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# PRACTICAL MECHANICS

AND

# ALLIED SUBJECTS

#### BY

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## PREFACE

This book together with that on "Practical Applied Mathematics" by the same author embodies the results of five years' experience in the organization and development of a system of railroad shop apprentice schools, as well as extended investigations of the work of public, private, trade, vocational and corporation schools of manufacturing industries and railroads in this country and abroad.

In the presentation no formal distinction is made of the various branches of elementary mathematics and science involved. The subject matter is presented in the two volumes under the general titles of "Practical Applied Mathematics" and "Practical Mechanics and Allied Subjects," although a careful and logical arrangement and sequence have been followed in the presentation.

In organizing trade apprentice schools difficulty has been experienced in securing a text practically adapted to the needs of these schools. This difficulty has led the author to develop a series of instruction sheets which are believed to be thoroughly practical. This volume with that on "Practical Applied Mathematics" is essentially a presentation of problems arising in shop experience in the mechanical trades. It shows the use of mathematics as a tool.

The material included in this work has been made sufficiently broad to apply not only to railroad schools but it is believed to schools in the mechanical trades generally. It is also felt that this work may be used to advantage in the public schools in technical branches, as well as in trade and vocational schools, either as a regular or as a supplementary text.

The material for each book is presented in twenty chapters, each dealing with a definite subject. This affords a flexibility highly desirable in assigning the work and enables it to be given entirely or in part according to specific needs.

In preparing this work the author acknowledges indebtedness to Pennsylvania Railroad Motive Power The author is indebted to Dr. J. P. Officials at Altoona. Jackson, State Commissioner of Labor and Industry of Pennsylvania, Professor J. A. Moyer, Head of the Department of Mechanical Engineering and Director of the Engineering Extension Division of the Pennsylvania State College and Mr. M. B. King, State Expert Assistant in Industrial Education of Pennsylvania, for reading the proofs and offering valuable criticisms and suggestions. He is especially indebted to Professor J. H. Yoder and Mr. E. W. Hughes of the School of Engineering of the Pennsylvania State College, detailed as Instructors in the Pennsylvania Railroad System of Apprentice Schools. They have assisted in the preparation of the manuscript, carefully read the proofs and offered many valuable suggestions.

JOSEPH W. L. HALE.

ALTOONA, PA. June 10, 1915

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# PRACTICAL MECHANICS AND ALLIED SUBJECTS

#### CHAPTER I

#### **FORCES**

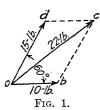
1. Definitions.—In our every day work we deal with forces of one kind or another. A pressure is a force. The earth exerts a force of attraction for all bodies or objects on its surface. Steam exerts a force or pressure in a locomotive boiler and cylinder. In grinding tools a force of friction has to be overcome. The wind exerts a force or pressure against the surface which it strikes.

When forces act on objects they move the objects if the forces are great enough to overcome the resistance to motion, and even if the objects do not move the forces produce a *stress* and *pressure* within and on the objects.

Forces are measured in pounds and tons. When we consider forces acting on objects we must know how the forces are applied, the direction of the forces and their values in pounds and tons. In working out problems forces are often represented by straight lines. The length of the line represents to scale the value of the force as 100 lb., 20 tons, etc., and the direction of the line, the direction in which the force acts as vertical, horizontal, etc.

2. Forces Passing through a Point.—In Fig. 1 the line Od 1 in. in length represents a force of 15 lb. and the line Ob 2/3 of an in. in length represents a force of 10 lb. Also force Ob is horizontal and force Od acts at

an angle of 60° with the horizontal.

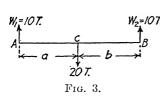


Two or more forces may be combined by the "parallelogram" method giving a singleforce having the same effect as the separate forces acting together, as for example in Fig. 1 forces of 10 lb. and 15 lb. acting on an object at 0 and 60° apart may be replaced by

a single force Oc of 22 lb. which is the same in effect as the forces of 10 lb. and 15 lb. acting together. The ingle force Oc is called the "resultant" of the forces Od and Oc In this Fig. lines Oc and c are parallel and equal, and the same is true of lines

Od and bc.

Rule.—The resultant of a number of forces is the direct sum of these forces only when they act in the same straight line, as in Fig. 2 and in the same direction.



Any force has the greatest  $W_2=107$  effect in the direction in which it acts, and no effect in a direction at right angles to that in which it acts.

3. Parallel Forces.—Parallel forces are those which act

in directions parallel to one another and which do not meet. In Fig. 3 is shown three forces, one of 20 tons and two of 10 tons each, balancing the *load* of 20 tons. For *parallel forces* as in this Fig.  $W_1 + W_2 = 20$ , and

$$\frac{W_1}{20} = \frac{b}{a+b}$$

and

$$\frac{W_2}{20} = \frac{a}{a+b}$$

#### PROBLEMS

1. What is the total force or pressure acting on a 14-in. piston in a steam cylinder with a steam pressure of 180 lb. per sq. in.?

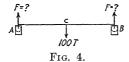
2. What must be the diameter of a piston in an air cylinder for a total pressure of 3000 lb. with a pressure per sq. in. of 60 lb.?

**3.** If the total pressure produced on a piston by steam in an 18-in. cylinder is 45,600 lb., what is the steam pressure in lb. per sq. in.?

4. What force of attraction does the earth produce on a steel rod 4 in. in diam, and 3 ft. 6 in. in length?

**5.** Two forces of 20 lb. and 30 lb. act on an object in opposite directions. What is the "resultant" force? If these forces acted at right angles to one another what would be the "resultant" force?

6. What is the total pressure produced on the supporting structure for a water tank if the tank is 12 ft. high and 8 ft. inside



diam. and full of water? The material of the tank weighs 1280 lb.

7. A traveling crane (Fig. 4) lifts a 100-ton locomotive. With the load half way between the crane supports A and B, what pressure is brought on each of these supports? How is this pressure changed when the load c moves nearer end A?

8. Using Fig. 4, find the pressures produced on supports at both A and B with the following loads and distances of the loads from A and B.

	Load at C	Distance AC	Distance BC	Pressure at A	Pressure at B
1 2 3 4 5	20 tons 15 tons 100 tons 2500 lb. 30 tons 390 lb.	20 ft. 32 ft. 10 ft. 50 ft. 42 ft. 25 ft.	40 ft. 28 ft. 50 ft. 10 ft. 18 ft. 35 ft.		

**9.** What steam pressure is used on locomotives? Is the same pressure used for heating cars and the shops?

Give complete answer with reasons.

10. If the steam pressure is 30 lb. per sq. in., what is the total force tending to blow the end out of a steam pipe 4 in. inside diam.?

#### CHAPTER II

# GRAVITATION. CENTER OF GRAVITY

4. Definition of Gravity.—The attractive force which every object in the universe has on every other object is called "gravitation" and this depends on the quantity of material composing the objects.

The force of attraction of the earth for objects is called the force of gravity and is what gives objects their "weight." A spring balance in giving the "weight" of any object measures merely the force of attraction of the earth for that object.

On account of "the force of gravity" which tends to pull objects toward the center of the earth work has to be done in lifting them and for the same reason elevated objects have power to do work on account of their elevated position, as in the case of water above a fall, elevated weights, train on a down grade, lifted hammer, etc. A locomotive hauling a train up a grade consumes more coal than when the train is on level track since work has to be done in lifting the weight of the train against the force of gravity as well as overcoming frictional resistances. A train descending a grade has work done on it by the force of gravity. The train will coast and often attain a high speed without the use of any steam.

5. Gravity Action On Freely Falling Objects. The "force of gravity" acting on objects free to fall increases their speed about 32.2 ft. per sec. each second of the

- fall. For example, a weight starting from rest and falling freely will have a speed of 96.6 ft. per sec. at the end of the third second of its fall and 161.0 ft. per sec. at the end of the fifth second of its fall.
- 6. Work Done Against Gravity.—The work done in lifting objects against "the force of gravity" is measured in *foot-pounds* and is equal to the weight of the object in pounds multiplied by the vertical height in feet through which the object is lifted.

#### **PROBLEMS**

- 1. A train is made up of a locomotive weighing 272,000 lb., a 158,000-lb. tender, a baggage and mail car weighing 86,000 lb. and 5 Pullman sleepers weighing together 625,000 lb. How much work is required to lift the train up a mile of grade rising 1 ft. in 250 ft.?
- 2. How much work in foot-pounds can a 2-ton hammer do in each fall if it is raised 16 ft.?
- **3.** How much work in foot-pounds is required to load the following lot of tires on a flat car, if the average distance through which they are raised is 6 ft.?

50 tires each weighing 452 lb. 37 tires each weighing 643 lb.

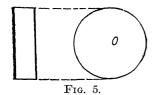
24 tires each weighing 598 lb.

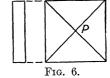
16 tires each weighing 544 lb.

- **4.** If a 10-ton casting is raised 23 ft. by a traveling crane, how much work is done? If one horsepower is the performance of 33,000 ft.-lb. of work per min. and the above casting is raised in 1 min., what horsepower does the motor on the crane exert?
- 5. Using the definition for "horsepower" given in Prob. 4: what work in foot-pounds could a 10-h.p. engine do in 5 min. if it ran at full capacity for this time?
- **6.** A single acting pump raises 1600 gal. of water every minute through a distance of 5 ft. How much work against gravity does the pump do per hr.?
  - 7. A train weighing 1000 tons descends 2 miles of grade dropping

1 ft. in 200 ft. What work in foot-pounds does "gravity" do on the train in the descent?

- 8. How much work is done per hr. in raising a 1-ton hammer through a vertical height of 4 ft. an average of fifteen times per min.?
- 9. If an object falls freely for 7 sec., what is its speed at the end of this time? Would a bullet and a feather fall side by side in the air? Has air friction any effect on the speed of a falling object?
- 7. Definition of Center of Gravity.—In the fourth paragraph we learned that the earth attracts toward its center every particle of every object on its surface, and this force of attraction called the force of gravity gives objects their weight. When solving problems in mechan-

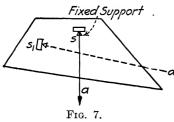




cs it is often necessary to find a point in an object at which its entire weight may be considered as concentrated. This point about which an object would balince is called its center of gravity. For example, a circular steel plate as in Fig. 5, has its "center of gravity" at the center 0, half way between the faces of the plate. If the inetal in this plate weighed .30 lb per cu. in. and the plate contained 100 cu. in. of metal we could consider to entire weight of .30 × 100 or 30 lb. as acting at 0, anstead of taking the weight of each cu. in. separately. This gives only one value, 30 to work with instead of 100 calues, each of .30.

The entire weight of a square uniform metal plate as in Fig. 6 may be taken at a point P at the intersection of the diagonals of the square and half way between the faces of the plate. The center of gravity of a cylinder is at a point on the axis of the cylinder and half way between the upper and lower bases.

The position of the center of gravity of an object such as a metal sheet, as shown in Fig. 7, can be found experimentally by using a plumb line and a support on which the object can move freely. When an object is suspended from a support on which it can move freely the



"center of weight" or "center of gravity" is brought as low as possible, and therefore lies in a vertical line drawn through the point of support. Using this principle an object as a steel plate shown in the figure is suspended freely from

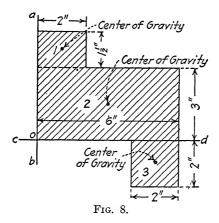
the support "s" and a plumb line dropped from the point of support as shown. The direction of the line is then marked on the plate giving line "sa."

The plate is next supported from a second point and with the plumb line we get line  $s_1a_1$ . The point where lines sa and  $s_1a_1$  cross marks the position of the "center of gravity" of the plate, half way between the front and back faces, if the plate is of uniform thickness throughout.

The center of gravity of sections made up of a number of simple sections as shown in Fig. 8, composed in this case of three rectangles, is found as follows: First we choose two "reference axes" as lines *ab* and *cd* crossing at 0. Next we find the *product* of the area of each simple

figure and the distance of *its* center of gravity from the chosen axis, then we add these products together and divide their sum by the sum of the areas of the simple figures.

For example, in the figure shown, the area of rectangle number 1 is  $2 \times 1\frac{1}{2}$  or 3 sq. in. The area of No. 2 is  $6 \times 3$  or 18 sq. in. and of No. 3,  $2 \times 2$  or 4 sq. in. The



sum of the products of these areas by the distances of their respective centers of gravity from axis ab gives  $3 \times 1 + 18 \times 3 + 4 \times 5$  or 77. Dividing this by the sum of the areas of the simple figures we have

$$\frac{77}{3+18+4}$$
 or  $\frac{77}{25}$  or 3.08 in.

which is the distance of the center of gravity of the whole figure from line or axis ab.

Working in the same way from axis cd we have the equation

Distance of c.g. from line

$$cd = \frac{3 \times 3.75 + 18 \times 1.5 - 4 \times 1}{3 + 18 + 4}$$

or  $\frac{34.25}{25}$  which equals 1.37 in.

The point where the lines cross as indicated in Fig. 9 is the position of the center of gravity of the figure or the point on which the whole plate would balance. In the equation above the last term in the numerator that

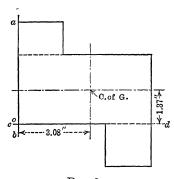


Fig. 9.

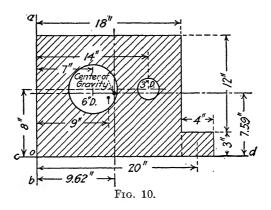
is  $4 \times 1$  has to be subtracted as shown rather than added since the area 4 sq. in. is on the *other side* of the reference axis cd, and if this was, for example, a metal plate we were calculating, the weight represented by the lower rectangle would help counterbalance the weight of the parts on the other side of the axis cd.

Example.—Find the center of gravity of the metal plate shown in Fig. 10 in which are cut two holes 3 in and 6 in, in diam.

Solution.—Distance of center of gravity from axis ab equals

$$\frac{18 \times 15 \times 9 + 4 \times 3 \times 20 - 6^2 \times .785 \times 7 - 3^2 \times .785 \times 14}{18 \times 15 + 4 \times 3 - 6^2 \times .785 - 3^2 \times .785}$$

or  $\frac{2373.3}{246.68}$  which equals 9.62 in.



Distance of center of gravity from axis cd equals

$$\frac{18 \times 15 \times 7.5 + 4 \times 3 \times 1.5 - 6^2 \times .785 \times 8 - 3^2 \times .785 \times 8}{18 \times 15 + 4 \times 3 - 6^2 \times .785 - 3^2 \times .785}$$

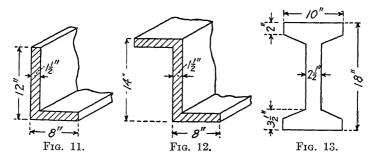
or  $\frac{1873.4}{246.68}$  which equals 7.59 in.

#### PROBLEMS

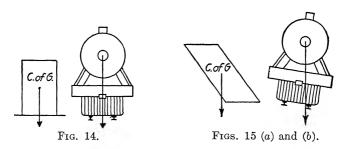
- 10. Find the center of gravity of the angle iron section shown in Fig. 11.
- 11. Find the center of gravity of the "Z" beam section shown in Fig. 12. If this beam is 10 ft. long, find its weight at .28 lb. per cu. in. for steel.
  - 12. Find the center of gravity of the I-beam section shown in

Fig. 13, and its weight if 18 ft. long and steel weighs .28 lb. per cu. in.

8. Stability of Objects.—Whether an object will stand up or fall over, that is, its "stability," depends upon



the position of its "center of gravity" with respect to its base. The rule is as follows: If a *vertical* line through the "center of gravity" passes inside the base of an object as in Fig. 14, the object will not fall, but if the



vertical line comes outside the base as in Fig. 15 $\alpha$ , the object will fall. From this it follows that objects which are heavy and have a large base are less likely to fall, than those which are lighter and have a smaller base,

and also the *lower* the "center of gravity" in an object the less likely it is to fall. The center of gravity of an "area" or "surface" of any kind is understood to mean that point about which a very thin sheet representing the surface would balance. Fig. 15b representing the end of a locomotive shows the change in position of the "center of gravity" with respect to the base when the locomotive is on a depressed rail on a curve.

#### **PROBLEMS**

- 13. A steel plate 2 ft. × 4 ft. is % in. thick. What is its weight and at what point in the plate can this entire weight be considered as concentrated? What is this point called?
  - 14. Where is the "center of gravity" of a sphere located?
- 15. At what point in a steel bar 4 in. in diam. and 6 ft. long can the entire weight be considered as acting? What is this weight?
- 16. Where is the center of gravity of a cube located? Find the weight and center of gravity of a cube of steel 2 in. on a side.
- 17. Why is it desirable to have the center of gravity of a pulley in line with the center line of the shaft on which the pulley works?
- 18. How could you find the center of gravity of any triangularly shaped piece of steel plate?
- 19. Is it necessary to use a counterbalance on a fly-wheel, and if so explain why?

#### CHAPTER III

#### DENSITY AND SPECIFIC GRAVITY

9. Definitions and Problems.—The density of an object means its mass or quantity of matter per unit of volume. Suppose a cubic foot of metal weighs 450 lb., we may then say that the density of the metal is 450 lb. per cu. ft. Since all objects are not of the same weight it is found convenient to have a ready means for calculating their relative weights. In order to do this, tables are prepared which give us the weights of unit volumes of materials compared with the weight of the same volume of some material taken as a standard.

Specific Gravity is the ratio of the density of two objects. For example: If a cubic foot of metal weighs 490 lb, and it is compared with the weight of a cubic foot of water which is about 62.5 lb., the ratio is  $\frac{490}{62.5}$  or 7.83 which is the specific gravity of the metal referred to water.

If a block of granite containing 1 cu, ft, volume weighs 162 lb., its weight compared to a cubic foot of water is  $\frac{162}{62.5}$  or 2.59 which is its specific gravity referred to water as a standard. In the same way we can compare the weights of other material, with water and prepare a table which we can use to advantage in calculations involving the weights of the material, most used.

In any standard handbook may be found uch table-

already prepared for a large number of substances. In making out these tables the weights of the bodies if they are solid or liquid are compared to the weight of the same volume of water as a standard, the weight of the water being taken as that at its temperature of greatest weight, namely, 4° Cent. If we are finding the relative weight of a gas we use hydrogen gas as a standard.

The specific gravity of a substance is therefore the ratio of the weight of a given volume of the substance to the weight of a similar volume of a substance taken as a standard. The specific gravity of a substance is therefore merely a number or a ratio and has no name attached to it.

If W is the weight of a substance, S the weight of the same volume of a standard substance and g the specific gravity of the substance we have the rule  $g = \frac{W}{S}$ . From this rule  $W = g \times S$ , that is, the weight of the substance is equal to its specific gravity times the weight of the same volume of the standard substance. Also we can write the equation in the form  $S = \frac{W}{g}$ .

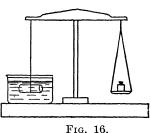
It is known that an object entirely immersed in a liquid loses in weight an amount equal to the weight of the volume of the liquid displaced by the object immersed. This principle enables us to determine easily the specific gravity of some substances by completely immersing them in liquids in which they are not soluble.

Example.—As shown in Fig. 16, a small cast-iron cylinder weighs in air 1.12 lb. When completely immersed in water as shown its weight is only .965 lb. The loss in weight of the object due to submerging it in

water is therefore .155 lb. and its specific gravity equals

 $\frac{\text{weight in air}}{\text{loss of weight when immersed in water}} = \frac{1.12 \text{ lb.}}{.155 \text{ lb.}} \text{ or } 7.22.$ 

Evidently we could not use this method with water as



a liquid if we wished to find the specific gravity of sugar, since sugar is soluble in water. In the case of substances which are dissolved by water we may use other liquids as the standard by which the substance is not dissolved and compare the weight of this new liquid with the weight

of water in determining the specific gravity.

Below is given a table of the approximate specific gravities of some of the most common materials.

#### APPROXIMATE SPECIFIC GRAVITIES

Iron, cast	7.21	Copper	8.79	Granite	2.6
Iron, wrought	7.78	Nickel	8.8	Glass	2.9
Steel	7.92	Mercury, at 60	°13.6	Pine, white	.55
Lead	11.3	Gold	19.26	Pine, yellow.	.66
Tin	7.29	Silver	10.47	Oak, white	.77
Zinc	7.19	Aluminum	2.67	Marble	2.7
Brass	7.82	Cork	.24	Ice	.917

Example.—Using the above table suppose we wish to find the weight of an iron casting containing  $2\frac{1}{2}$  cu. ft.

Solution.—One cubic foot of water weighs 62.5 lb. The specific gravity of cast iron is 7.21, that is, the cast iron is 7.21 times heavier than water, therefore the weight of the casting equals

 $62.5 \times 2.5 \times 7.21$  or 1126.56 lb.

Example.—What is the weight of a white-oak beam  $4 \text{ in.} \times 8 \text{ in.}$  and 18 ft. long?

Solution.—We first find the volume of the beam in cubic feet which is equal to  $\frac{4}{12} \times \frac{8}{12} \times 18$  or 4.0 cu. ft.

Since from the table we see that the specific gravity of white oak is .77 that is, since it is .77 times as heavy as water the weight of the beam equals

cubic feet 
$$\times$$
 62.5  $\times$  .77

that is  $4.0 \times 62.5 \times .77$  or 192.5 lb.

Example.—A timber is thrown into still water and it is found that it floats one-third out of water, what is its specific gravity?

Solution.—Since the timber displaces a volume of water equal to  $^23$  of its own volume it is held up or buoyed up by a force equal to the weight of a volume of water  $^23$  that of the volume of the timber. The weight of the timber is therefore only  $^23$  of the weight of the same volume of water, that is, its specific gravity is .67.

Example. What is the volume of 100 lb. of copper?

Solution: One cubic inch of water weighs  $\frac{62.5}{1728}$  or .0361 lb. Since the specific gravity of copper is 8.79, 1 cu. in. of copper weighs  $8.79 \times .0361$  or .317 lb. There are therefore as many cubic inches of copper in 100 lb. as .317 lb. the weight of 1 cu. in. of copper is contained into the total weight of copper or  $\frac{100}{.317} = 315.4$  cu. in.

#### PROBLEMS

 Using the table of specific gravities, find the weight of 8 cu, ft, of wrought iron, 2. Find the weight of a white pine timber 4 in. .- 6 m. and 14 ft. long.

3. How many pounds heavier is a cubic foot of lead than a

cubic foot of wrought iron?

4. Find the number of cubic feet in a marble block weighing 2½ tons.

**5.** A cylindrical casting  $2\frac{1}{2}$  in, in diam, and 8 in, long weighs

11.0 lb. What is its specific gravity?

- 6. A tank car has a capacity of 50,000 lb. If it is to be filled with petroleum having a specific gravity of .832, how many cubic feet and how many gallons can the tank hold?
- 7. The pressure of the atmosphere per square inch is about 14.7 lb. How high a column of mercury 1 sq. in, in cross section is required to produce this pressure?

8. A steel casting displaces 4.84 gal. of water. What is its weight?

9. Two hundred cubic inches of brass are made up of 150 parts of copper and 50 parts of zine by volume. Find its specific gravity. Solution:

The weight of 1 cu. in. of water = .0361 lb.

The weight of 1 cu. in. of copper =  $8.79 \times .0361$  or 317 lb Therefore 150 cu. in. weigh 47.6 lb. The weight of 1 cu. in. of zinc =  $7.19 \times .0361$  or .260 lb. Therefore 50 cu. in. weigh 13.0 lb.

If S is the specific gravity of the brass, 200 cu, in, weigh

$$200\times.0361\times\mathcal{S}$$
 lb.

Since the weight of the brass equals the sum of the weights of its parts, we have,  $200 \times .0361 \times S = 47.6 \times 13$  therefore

$$S = \frac{47.6 + 13}{200 \times .0361}$$
 or 8.39 the specific gravity

- 10. Forty pounds of copper and 27 lb, of zine are alleged to make brass. What is its specific gravity?
- 11. A piece of metal weighs 26.8 lb, in air and 2.34 lb, when entirely immersed in water. What is its specific gravity?
- 12. Find the weight of oil in a cylindrical tank 8 ft in dram and 16 ft, long if the specific gravity of the oil is 98.

- 13. A casting weighs in air 22.8 lb. When entirely immersed in oil whose specific gravity is .94 the easting weighs 19.6 lb. Find its specific gravity and its volume in cubic inches.
- 14. An alloy is made up of 30 cu. in. of tin and 48 cu. in. of copper. Find its specific gravity.

#### CHAPTER IV

#### SCREW THREADS

10. Thread Sections.—The shape and size of a thread are determined by the shape and size of its cross section taken in a plane passing through the axis of the screw. The dimensions of any particular kind of thread are fixed by stating the number of threads or sections in an inch, measured parallel to the axis of the screw, as, for example, 8 threads per inch, or simply 8 threads.

Fig. 17 shows the sections of four different screw threads. In the machine shop on regular work the U.S.



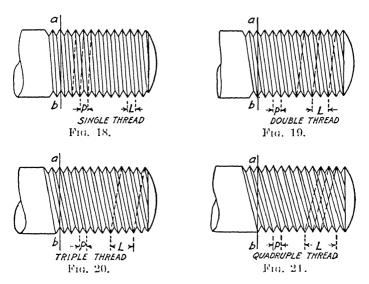
Fig. 17.

Std. Thread is used. In some practice bolts and studs 34 in. in diam, and above serewed into boilers have twelve sharp V threads per in., irrespective of the diameter of the bolt or stud.

11. Pitch and Lead.—Before looking into the method of constructing and calculating threads we should understand first the meaning of the "pitch" and "lead" of a screw. The "pitch" of a screw is the distance from the center of one thread to the center of the next. The "lead" of a screw is the distance the screw advances in one complete turn.

12. Single, Double, and Triple Threads.—A single-threaded screw, as shown in Fig. 18, is one which advances in one complete turn a distance (measured in the direction of the axis of the screw) equal to that from a point on one thread to a corresponding point on the next thread, or a distance equal to the "pitch."

A double-threaded serew, as shown in Fig. 19, ad-

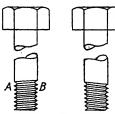


vances in one complete turn a distance equal to twice the "pitch." For a double thread the "lead" is therefore twice the "pitch." In a triple thread, as shown in Fig. 20, the advance for one complete turn equals three times the "pitch," or the "lead" equals three times the "pitch." Fig. 21 shows a quadruple thread.

13. Right- and Left-handed Threads. A right-handed thread is one in which the thread rises from left to right,

as from A to B in Fig. 22. Fig. 23 shows a left-handed thread.

Suppose now, for example, we were making a sketch of a threaded bolt, we should show the following. The total length of bolt including the head, the length of



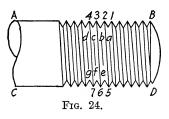
Frgs. 22 and 23.

body including the threaded portion, length of threaded portion, diameter of bolt, the thread section, number of threads per in., whether thread is right- or left-handed and whether single, double or triple. If there are no standard tables available for finding the dimensions of the bolt

head, the thickness and short diameter of the head should be shown.

14. Construction of a Single, Right-handed V Thread, 8 Threads to an Inch.—Lay off on line AB, Fig. 24 (an element of the portion to be threaded),  $\frac{1}{8}$ -in. distances as

1-2, 2-3, etc., each equal to the pitch of the screw. With the  $60^{\circ}$  triangle draw the sections 1a2, 2b3, etc. A line through point a parallel to line AB contains the points b, c, d, etc., at the bottoms of the threads. A perpendicular

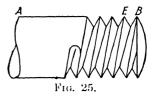


from point a to line CD gives point 5 on the *outside* of the thread. Similarly perpendiculars dropped from points b, c, d, etc., locate points on the outside of the thread on line CD. With the 60° triangle the sections 5e6, etc., are completed and the visible edges of the thread are drawn by connecting points b and e, 2 and 5, etc., as

shown. The edges of the thread form turns called "helices," a single turn being a "helix." These edges are, however, simply represented by straight lines which are used in most cases as in Fig. 24.

15. Construction of a Double, Left-handed V Thread, 4 Threads to an Inch.—In Fig. 25 portion AB is to be threaded. The "pitch" is here ¼ in., and the "lead" ½ in. Since the thread is double, the outside points of the thread on opposite sides of the serew fall opposite each other. The construction is otherwise the same as for the single V thread.

In one complete turn the screw advances a distance equal to twice the "pitch," or a distance of  $\frac{1}{2}$  in, equal to the "lead" of the screw. The distance BE corresponds to the pitch. The



screw is left-handed, since the thread ascends from right to left looking toward the head of the bolt.

16. Construction of a United States Standard Thread. The U. S. Std. for screws and threads is that adopted by the U. S. Government and by the R. R. Master Mechanics and Master Car Builders Associations, also by leading tool builders and manufacturers. This thread has the same angle as the V thread or  $60^{\circ}$  thread. The dimensions of a thread for a pitch equal to 1 in. are shown in Fig. 26. The triangle ABC is equilateral. If each of its sides equals 1 in. the distance CD equals the square root of  $[(1)^2 + ({}^12)^2]$  or the square root of (1 + 0.25) or 0.866 in. This is therefore the depth of a V thread of pitch equal to one. In this U. S. Std. thread the depth EF = 0.65 in., that is, for a pitch of 1 in. The distance across the flats at top and bottom is 0.125

in., as shown in the figure, that is,  $\frac{1}{8}$  of the pitch. To find the depth of a U.S. Std. thread of another pitch, divide these figures by the number of threads to the inch.

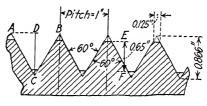


Fig. 26.

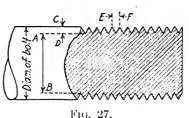
#### **PROBLEMS**

- 1. What is the lead of a single-threaded screw having 12 threads per in.? What is the pitch of this screw?
- 2. A double thread measures .125 in. between the points, what is its pitch and lead?
- 3. How many turns must a single thread of pitch .166 in. make to advance 11/4 in.?
- **4.** A double-threaded screw advances  $1\frac{1}{2}$  in. in three complete turns. What is its pitch?
- 5. Find the depth of thread, and width of flats for a U. S. Std. Thread, with the following number of threads per in. 8, 12, 13, 18.
- 6. The number of threads per inch (U. S. Std.) corresponding to the diameter of the bolt are as follows:—Diam. of bolt ½ in.; threads per in., 13; diam. ½ in.; threads, 9; 1½ in. diam. threads, 7. For each of these cases calculate the diameter at root of thread and the area of bolt at root of thread, also the width of flat.
- 7. In a machine, three bolts each 1 in. in diam. with 8 U. S. Std. threads per in., are replaced by a single bolt. What must be the size of this bolt at root of thread to have the same area of metal at root of thread as the three 1-in. bolts?
- 8. What is the "pitch" and the "lead" of a triple thread which advances 2 in. in two complete turns?
- 9. Make a sketch of a double, right-hand V thread with a pitch of ½ in.

- 10. Find the size of a tap drill in nearest 64ths in, for a "V" thread 8 per in, on a bolt 1 in, in diam.
- 11. Make a sketch of a single right-hand U. S. Std. thread with 8 threads per in. What is the pitch and the lead in this case?
- 12. How does the number of threads per in, on a bolt and hence the depth of the thread affect the strength of the bolt? Would it be practicable to have only 4 threads per in, on a 1-in, bolt? Why with some companies are 12 sharp V threads per in, used in boiler work on any size bolt or stud above  $\frac{3}{4}$  in, in diam,?
- 13. What is the advantage of double and triple threads over a single thread? How does the number of threads per in, affect the "speed" of a bolt or screw? Which form of thread is used most in the machine-shop work, the V or U. S. Std.?

Referring to Fig. 27, showing the section of a V thread, the distance AB, or the diameter at the root of the thread

equals the diameter of the bolt minus twice the distance CD. The length CD is .86 of the distance EF, and the distance EF depends on the number of threads per in, of the screw. In an 8-thread



serew  $EF={}^{-1}\mathbf{s}$  in, or .125 in., for a 10-thread serew EF=

 ${}^{1}_{10}$  in, or .10 in., for a T-thread, serew  $EF = \frac{1}{T}$ 

From the above we have the rule for a V thread.

$$R = D \sim \frac{1.73}{T}$$

where R Diam, at root of thread.

D Diam, of bolt (outside of threads).

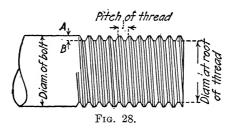
T No. of teeth or threads per in.

If in the case of bolts or study  $^3{}_4$  in, in diam, and above, screwed into boilers, 12 sharp V threads per in, are used,

of the diameter of the bolt or stud, the prebecomes in this case:

$$R = D - \frac{1.73}{12}$$
 or  $R = D - 0.144$  in.  
or  $R = D - \frac{9}{64}$  in.

we of the U. S. Std. thread as in Fig. 28 the at the root of the thread equals the diameter t minus twice the distance AB. The distance AB the pitch. Therefore the diameter at the bethread



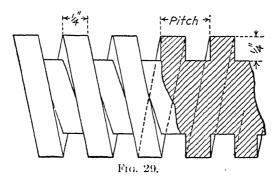
- = diameter of bolt  $-2 \times 0.65 \times$  the pitch.
  - The pitch =  $\frac{1}{T}$  where T is the number of threads per in.

Therefore diameter at root of thread for U. S. Std.

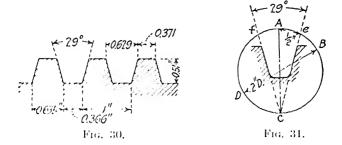
- = diameter of bolt  $-\frac{1.30}{T}$
- 17. Construction of a Single, Square Thread, 2 Threads Per Inch. Fig. 29 Shows the Method of Construction.
- 18. Construction of an Acme Thread.—Square threads are not always easily cut, and there has been developed a standard, which is flat at top and bottom and has its sides inclined at an angle of  $14\frac{1}{2}^{\circ}$  to the center line of

the thread section. Fig. 30 shows the section for such a thread and the dimensions of the threads for a pitch of 1 in.

The angle of 29° or the total angle formed by the sides



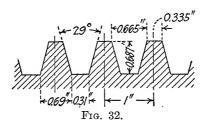
of an Acme thread as in Fig. 31 may be drawn with the use of a protractor or as shown in the figure which is drawn as follows:



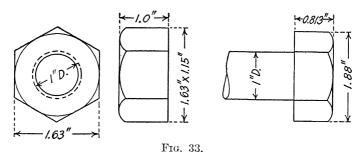
A circle ABCD is drawn 2 in, in diam, and also a vertical diam, AC. From A as a center and with a radius of  $^{1}_{2}$  in., ares are drawn cutting the circle at points c and f. Connecting these points with point C by the straight

lines eC and fC gives the inclination for the sides of the threads. The angle eCf is  $29^{\circ}$ .

In a "Worm" thread as in Fig. 32 the angle is the same as for the Acme thread, that is, 29°, but the thread is about one-third (1/3) deeper than the Acme.



19. Miscellaneous Rules.—The width across flats of U. S. Std. hex. nuts equals  $1\frac{1}{2} \times \text{Diameter of bolt} + \frac{1}{8}$  in. See Fig. 33. The width across corners of U. S. Std. hex. nuts equals  $1.155 \times \text{width across flats}$ . The



thickness of U. S. Std. hex. nuts (rough) is the same as diameter of bolt. Width across corners and flats of bolt heads is the same as for hex. nuts. The thickness of bolt heads equals half the width across flats of bolt head or  $34 \times \text{diameter}$  of bolt  $+ \frac{1}{16}$  in.

TABLE FOR U.S. STD. THREADS

Diam. of screw	Threads to inch	Pitch	Depth of thread	Diam. at root of thread	Width of flat
1/4	20	. 0500	.0325	.185	. 0063
5/16	18	. 0556	.0361	.2403	. 0069
3/8	16	. 0625	.0405	.2936	.0078
7/16	14	.0714	. 0461	.3447	.0089
1/2	13	.0769	.0499	.4001	.0096
916	12	. 0833	.0541	.4542	.0104
5/8	11	.0909	. 0591	.5069	.0114
$\frac{3}{4}$	10	. 1000	.0649	.6201	.0125
7/8	9	.1111	.0721	.7307	.0139
1	8	. 1250	.0812	.8376	. 0156
11/8	7	. 1429	.0928	.9394	.0179
11/4	7	. 1429	.0928	1.0644	.0179
$1\frac{3}{8}$	6	. 1667	.1082	1.1585	.0208
$1\frac{1}{2}$	6	. 1667	.1082	1.2835	.0208
158	51/2	. 1818	.1181	1.3888	.0227
134	5	. 2000	.1299	1.4902	.0250
17/8	5	. 2000	. 1299	1.6152	.0250
2	41/2	. 2222	. 1444	1.7113	.0278
$2\frac{1}{4}$	412	.2222	. 1444	1.9613	.0278
$2\frac{1}{2}$	4	. 2500	. 1624	2.1752	.0313
23/4	4	.2500	. 1624	2.4252	. 0313
3	312	.2857	. 1856	2.6288	.0357
31/4	312	.2857	. 1856	2.8788	.0357
$3\frac{1}{2}$	3,14	. 3077	. 1998	3.1003	.0385
334	3	. 3333	.2165	3.3170	.0417
4	3	. 3333	.2165	3.5670	.0417
414	27/8	.3478	. 2259	3.7982	.0435
41/2	$2^3$	. 3636	. 2362	4.0276	.0155
434	$2^{5}_{28}$	.3810	. 2474	4.2551	.0476
5	$2\frac{1}{2}$	.4000	.2598	4.4804	.0500
514	$2\frac{1}{2}$	.4000	.2598	4.7304	.0500
$\frac{512}{2}$	238	.4210	.2735	4.9530	.0526
534	238	.4210	.2735	5.2030	.0526
6	214	.4444	.2882	5.4226	.0556

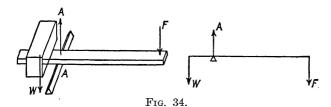
TABLD FOR SHARP V THREADS

			,	<del></del>
Diam. of screw	No. threads per inch	Pitch	Depth of thread	Diam. at root of thread
$\frac{1}{4}$	20	.0500	.0433	.1634
5∕16	18	. 0556	.0481	. 2163
3/8	16	.0625	.0541	. 2667
<b>½</b> 16	14	.0714	.0618	. 3140
$\frac{1}{2}$	12	.0833	.0722	. 3557
%16	12	.0833	.0722	. 4182
5⁄8	11	. 0909	.0787	. 4676
<sup>1</sup> ½6	11	.0909	.0787	. 5301
$\frac{3}{4}$	10	.1000	.0866	. 5768
<sup>1</sup> 3⁄16	10	.1000	.0866	. 6393
7/8	9	.1111	.0962	. 6826
<sup>1</sup> 5⁄16	9	.1111	.0962	.7451
1	8	. 1250	.1083	. 7835
11/8	7	. 1429	.1237	. 8776
$1\frac{1}{4}$	7	.1429	.1237	1.0026
$1\frac{3}{8}$	6	. 1667	.1443	1.0864
$1\frac{1}{2}$	6	. 1667	. 1443	1.2114
$1\frac{5}{8}$	5	. 2000	.1733	1.2784
$1\frac{3}{4}$	5	. 2000	. 1733	1.4034
$1\frac{7}{8}$	$4\frac{1}{2}$	. 2222	.1924	1.4902
2	$4\frac{1}{2}$	.2222	. 1924	1.6152
$2\frac{1}{8}$	$4\frac{1}{2}$	.2222	.1924	1.7402
$2\frac{1}{4}$	41/2	. 2222	.1924	1.8652
23/8	$4\frac{1}{2}$	. 2222	. 1924	1.9902
$2\frac{1}{2}$	4	.2500	.2165	2.0670
25/8	4	.2500	.2165	2.1920
$2\frac{3}{4}$	4	. 2500	.2165	2.3170
27/8	4	.2500	.2165	2.4420
33	3½	. 2857	.2474	2.5052
1/8	3½	. 2857	.2474	2.6301
$3\frac{1}{4}$	3½	. 2857	.2474	2.7551
33/8	$3\frac{1}{4}$	. 3077	.2666	2.8418
$3\frac{1}{2}$	31/4	.3077	.2666	2.9668
35/8	31/4	. 3077	.2666	3.0918
33/4	3	. 3333	.2886	3.1727
37/8	3	. 3333	.2886	3.2977
4	3	. 3333	.2886	3.4227

## CHAPTER V

## CALCULATION OF LEVERS

20. Definitions and Problems.—A lever is a rod, bar or beam of any shape, either straight or curved, which is free to turn about a fixed point called a "fulcrum." The fulcrum may be any form of support, prop or bearing.



Levers are divided into three classes according to the positions of the loads, pressures, or forces acting on the lever with respect to the position of the fulcrum. In

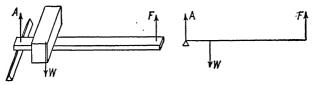


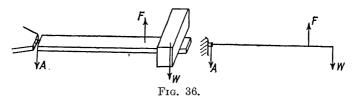
Fig. 35.

the first class, Fig. 34 the fulcrum A is between the weight (W) and the force (F) applied to lift the weight.

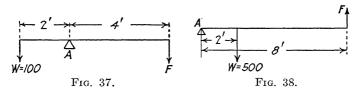
In the second-class lever Fig. 35 the weight W is between the fulcrum A and the force F.

In the third-class lever the force F is between the fulcrum A and the weight W, as in Fig. 36.

In working out lever problems we find what are called the "moments" of the forces or pressures or weights acting on the lever. In Fig. 37 which shows a lever of the first class the force F, 4 ft. from the fulcrum A,



raises the weight W of 100 lb. located 2 ft. from A. Multiplying F by 4 lb., its distance from A gives the moment of F about the "fulcrum" A. Similarly  $W \times 2$  ft. is the moment of W about the "fulcrum" A. To find what force is necessary at F to raise a weight of 100 lb. at W, we multiply  $F \times 4$  ft. giving the moment of

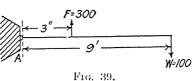


F about A and place this product equal to  $100 \times 2$  which is the moment of the weight of 100 lb. about A. Then

therefore 
$$100 \times 2 = F \times 4$$
 
$$200 = 4F$$
 and 
$$F = \frac{200}{4} \text{ or } 50 \text{ lb.}$$

Therefore 50 lb. pull at F will lift 100 lb. at W neglecting friction and the weight of the lever itself. The rule in any case is, find the moment of the force about the

fulcrum and place this equal to the moment of the weight about the fulcrum and solve for the quantity desired.



In Fig. 38 to find a

force F necessary to lift a weight of 500 lb. at the distance given we have  $500 \times 2 = F \times 8$  therefore

$$1000 = 8F$$
  
 $F = 125 \text{ lb.}$ 

and

In Fig. 39 which represents a lever of the third class a force of 300 lb. at F will lift only 100 lb. at W, since taking the "moments" about the fulcrum A we have  $300 \times 3 = W \times 9$ .

Therefore and

$$900 = 9W$$
  
 $W = 100 \text{ lb.}$ 

## **PROBLEMS**

What class of lever does Fig. 40 show and why? From the rules already given and using Fig. 40, fill out the following table,



Fra. 40.

giving also sample calculations to show how you obtained your results. Neglect the weight of the lever itself.

No.	W, lb.	F, lb.	а	ь
1	30		1 ft.	3 ft.
<b>2</b>		50	2 ft.	8 ft.
3	60	30		6 ft.
4	80	10	2 ft.	
5	18		3½ ft.	$4\frac{3}{4}$ ft.
6		40	8 in.	2 ft.

What class of lever does Fig. 41 show and why? Using Fig. 41 fill out the following table, giving also sample calculations to

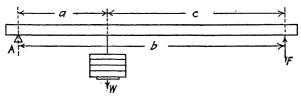


Fig. 41.

show how you obtained your results. Neglect the weight of the lever itself.

No.	Flb.	W lb.	а	b	с
7 8 9	50 30 30	200 10 60	8 ft. 6 ft. 1 ft.	5 ft. 4 ft.	2 ft.
11 12	27	20	6½ ft.	39 in.	2 ft. 27 in.

What class of lever does Fig. 42 show, and why? Using Fig. 42 fill out the following table, giving also sample calculations to show how you obtained your results. Neglect the weight of the lever itself.

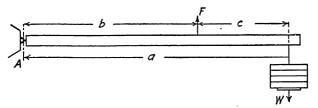
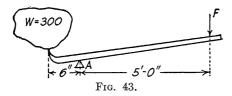


Fig. 42.

No.	W, lb.	F, lb.	a	ь	c
13	50	200	8 ft.		
14	30		6 ft.		2 ft.
15		10	10 ft.	5 ft.	
16	30	60		4 ft.	
17		20	$6\frac{1}{2}$ ft.		2 ft.
18	27			39 in.	27 in.

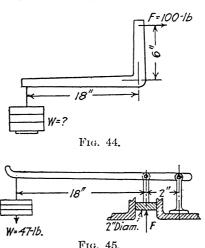
19. As shown in Fig. 43, what downward force at F is necessary to start the stone if the resistance offered is 300 lb. If the stone



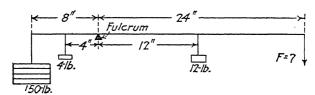
could not be moved with the bar as shown how could a greater force or pressure be produced on the stone, using the same bar? What class of lever does this represent?

- 20. In Fig. 44 the lever arms are bent at right angles to each other, allowing the effect of the weight W to be produced in a horizontal direction at F. With the lever arms as shown, what weight is necessary at W to produce a pull of 100 lb. at F?
  - 21. In Fig. 45, showing a safety valve for a stationary boiler,

at what pressure will the boiler "blow off" with the size of valve and lever arms as shown? What class of lever does this show?



21. Calculation for Levers Taking into Account the Weight of the Lever Itself in Each Case.—If the lever shown in Fig. 46 weighs .5 lb. per in. of length the portion to the left of the fulcrum weighs  $8 \times .5$  or 4 lb.

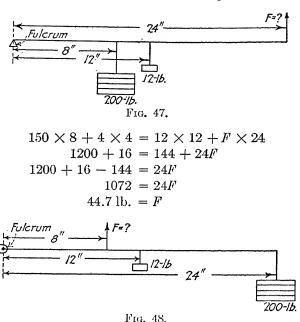


Fro. 46.

The portion to the right of the fulcrum weighs  $24 \times .5$  or 12 lb. These weights should be taken at the *center of gravity* of the respective parts of the lever.

For example, the entire weight of the 8-in. length is

considered as concentrated at a point 4 in. from the fulcrum, that is, at the center of gravity of the 8-in. length. Likewise the 12 lb. is considered as concentrated at a distance of 12 in. from and at the right of the fulcrum. We now have four forces to deal with and our equation becomes:



In Fig. 47, the lever weighs .5 lb. per in. of length or 12 lb. total weight. This acts at a distance of 12 in. from the fulcrum, that is, at the *center of gravity* of the whole lever. The equation then becomes

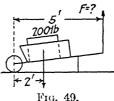
$$200 \times 8 + 12 \times 12 = F \times 24$$
  
 $1744 = 24F$   
 $72.6 \text{ lb.} = F$ 

In Fig. 48, the lever weighs 12 lb. and the equation becomes

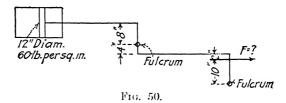
$$12 \times 12 + 200 \times 24 = 8F$$
  
 $4944 = 8F$   
 $618 \text{ lb.} = F$ 

#### PROBLEMS

- 22. In a lever of the first class a weight of 250 lb. 6 in, from the fulcrum can be balanced by what force applied 11<sub>2</sub> ft. from the fulcrum?
- 23. Work out the above problem if the lever itself weighs 4 lb, per ft. of length.

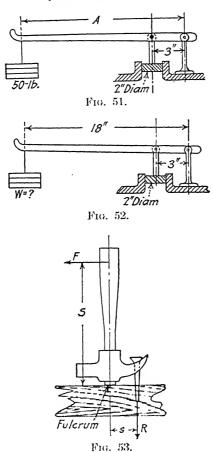


- **24.** Find the force F required to lift the weight of 200 lb, on the wheelbarrow as shown in Fig. 49.
- 25. From Fig. 50 find the force which can be exerted at F through the air cylinder and system of levers shown.



**26.** From Fig. 51, showing a safety valve for a stationary boiler, what must be the length of the arm "A" in order that the boiler will "blow off" at 150 lb. pressure per sq. in.? What class of lever does this sketch show?

27. From Fig. 52, showing a safety valve for a stationary boiler, what weight must be placed at W in order that the boiler will "blow off" at 150 lb. pressure per sq. in.?



28. In using an ordinary claw hammer for pulling a nail as shown in Fig. 53, we are using a first-class lever. Suppose as shown

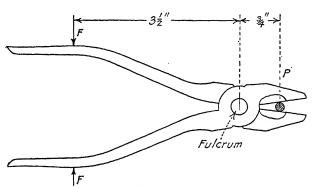


Fig. 54.

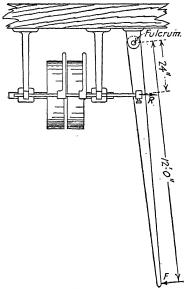
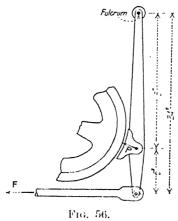


Fig 55.

a force of 40 lb. is exerted at "F," we find the resistance of the nail at the start by calculating a first-class lever, whose arms are bent up at right angles to each other. If the distance s=2 in. and S=9 in., calculate the resistance "R" of the nail at the start.

- 29. The pliers and wire cutters shown in Fig. 54 form a first-class lever. If the total force on the handle of the pliers is 10 lb. at a distance of  $3\frac{1}{2}$  in from the fulcrum, how much pressure is produced on the wire at "P" in cutting it? The wire is  $\frac{3}{4}$  in from the fulcrum.
- **30.** Fig. 55 shows a "belt shifter." If the resistance at "R," required to be overcome in shifting the belt is 35 lb., what force is required at "F" to move the belt with the lever arms shown?



**31.** If in shifting the belt it is necessary to move the shifter rod 4 in, how far does the end of the long arm move, that is, the end on which the force "F" is exerted?

In Fig. 56 is shown a second-class brake lever with brake shoe and brake rod. The motion of the brake shoe is to the motion of the pull rod as 18:24 with the lengths of lever arms as shown. For example if the

brake rod moves 4 in. the shoe movement is found as follows:

$$\frac{\text{Movement of brake shoe}}{\text{Movement of pull rod}} = \frac{18}{24} \text{ or } \frac{\text{movement of brake shoe}}{4}$$
$$= \frac{18}{24}$$

That is, movement of brake shoe =  $4 \times \frac{18}{24} = 3$  in.

Also if the braking force through the brake shoe is called "P" we have the proportion  $\frac{P}{F} = \frac{24}{18}$  from which

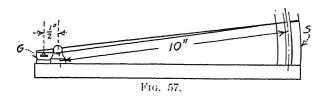
$$P = \frac{F \times 24}{18}.$$

Example.—If the pull on the rod is 4800 lb., that is, if F = 4800 lb. then  $P = \frac{4800 \times 24}{18}$  or 6400 lb.

- **32.** In the brake lever shown in Fig. 56, if the distance from the fulcrum to brake shoe is 24 in. and the whole effective length of lever is 30 in, what is the movement of brake shoe compared with that of the brake rod? What pull is necessary to produce a pressure of 8000 lb. against the wheel?
- **33.** Find the length of lever arms for a brake lever as shown in Fig. 56 such that a pull of 800 lb. on the brake rod will produce a pressure of 4000 lb. of the brake shoe against the wheel. In this case what is the ratio of movement of brake shoe and brake rod?

Levers used to Increase or Decrease Motion. --Levers are often used to increase motion which of itself is too small to read easily as in the case of an indicator as shown in Fig. 57. With this instrument the piece to be measured is put in at "G." The instrument is "calibrated" or compared with a standard gauge to find the reading on the scale "S" for a given gauge. When the piece is put in at "G" any movement of the pointer above or below the gauge point on "S" shows how much too large or too

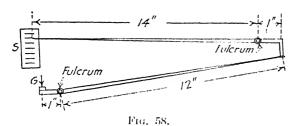
small the piece is that is being measured. The reading of the scale for different pieces tested will depend on the ratio of the arms of the lever used. If the short arm is .5 in. and the long arm 10 in. as shown the ratio of the motion of the ends of the lever is as 1:20. If for



example in testing a piece the end of the long lever moves .2 in. beyond the standard gauge mark, the work being measured is one-twentieth of .2 or .01 in. from the standard.

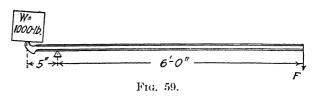
#### PROBLEMS

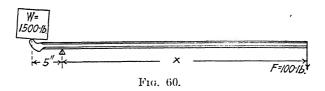
**34.** In the indicator shown in Fig. 57 the pointer indicates on scale "S" a distance of .48 in, beyond the gauge mark. How much is the piece being measured out of gauge?

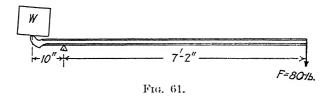


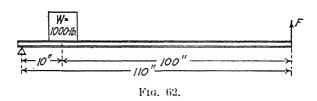
**35.** An indicator similar to that shown in Fig. 57 has lever arms of .75 in, and 18 in. If a piece being measured is .025 in, from standard gauge, how far will the pointer move on scale "S" from the standard gauge mark?

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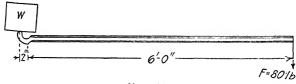
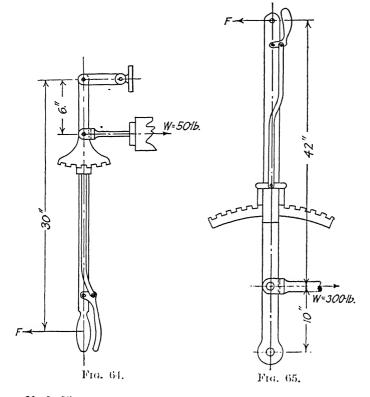


Fig. 63.

- **36.** In the lever indicator system shown in Fig. 58, find the motion produced at "S" by a motion of .040 in. at G.
- 37. Show how you could redesign the levers so that a motion of .040 in. at G would produce at "S" twice the motion calculated in the problem above.



- 38. In Fig. 59, find the force F necessary to raise the weight W.
- **39.** In Fig. 60, find the length of the lever arm X so that the force of 100 lb, will balance the load W of 1500 lb.
- **40.** In Fig. 61, find the weight W that can be raised by the force of 80 lb.

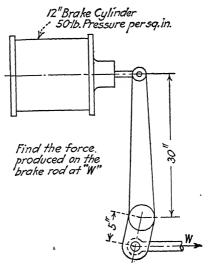


Fig. 66.

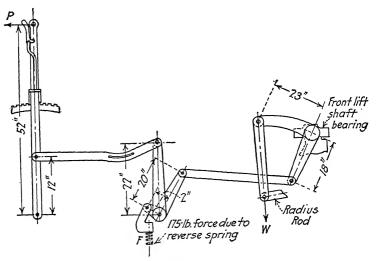


Fig. 67.

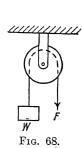
- **41.** In Fig. 62, find the force F required to raise the weight W.
- 42. In Fig. 63, showing a pinch bar 6 ft. long with 2-in. nose, find the weight W that can be raised.
- 43. In Fig. 64, showing a throttle lever, find the force F required to move the throttle.
- 44. Fig. 65 shows a reverse lever. Find the force F required to reverse the engine.
- **45.** Fig. 66 shows a brake cylinder. Find the force produced on the brake rod at W by the pressure of 50 lb. per sq. in. in the cylinder.
- **46.** With a load at W, Fig. 67, of 400 lb. due to lifting the link blocks, radius rod ends and moving the valves, what will be the force required at P to move the reverse lever backward taking into account the action of the reverse spring, but neglecting the friction of the reversing gear?

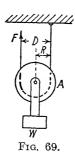
## CHAPTER VI

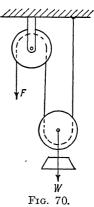
# PULLEYS (BLOCK AND TACKLE)

22. Arrangement of Pulley Systems.—Pulleys or blocks (Fig. 68) are used to change the direction of application of a force, as well as to reduce the value of the force necessary to lift a given weight or overcome a given resistance. Pulleys are in fact rotating levers.

With a single pulley as in Fig. 68, the force F applied to raise the weight W, can be exerted downward instead of upward. This makes it easier to







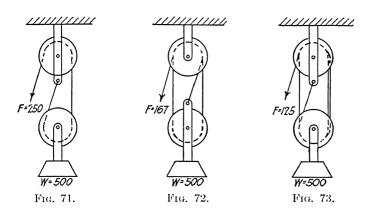
raise the weight. In this case the force required at F is equal to the weight lifted at W, neglecting friction.

This arrangement forms a *lever* of the first class. Compare this with a first-class lever.

In the case of a single movable pulley, as in Fig. 69, the force necessary at F is one-half the weight lifted at

W, neglecting the effect of friction. This arrangement forms a lever of the second class and we have the weight  $W \times \text{radius}$  of pulley  $= F \times \text{diameter}$  of pulley. Therefore to lift 100 lb. at W requires a force of 50 lb. at F. Compare this figure with a second-class lever.

Fig. 70 shows a fixed and a movable pulley, or a combination of the arrangements shown in Figs. 68 and 69. The force required at F to lift a weight W is



equal to one-half of the weight neglecting friction of the rope and pulleys. When the force F is exerted through a distance of 2 ft., the weight is lifted 1 ft. A force of 50 lb. at F will raise a weight of 100 lb. at W. The friction of the rope on the pulleys and the pulleys on their bearings is usually small compared to the weights ordinarily raised, and this friction is reduced by the use of grease or oil on the bearings.

In the following arrangements of blocks, Figs. 71, 72, and 73, the force required to raise a given weight

or vice versa is found as follows: Rule.—The force in pounds multiplied by the distance in feet through which it moves, equals the weight in pounds lifted, multiplied by the distance through which it moves, and another rule which also applies for any number of pulleys in either block is: Divide the weight to be lifted by the number of single strands or lines of rope extending down to the movable pulley, to find the force F. The distance through which the weight moves equals the distance through which the force F moves divided by the number of single lines of rope extending down to the movable block. In Fig. 71 a force of 250 lb. is necessary to lift a weight of 500 lb., neglecting friction. For each of the arrangements shown the force necessary to raise a weight of 500 lb. is given.

#### PROBLEMS

- 1. With a single movable pulley as in Fig. 69, what force is necessary at F to raise a weight of 300 lb.? What advantage has the arrangement shown in Fig. 70 over this one using only the single movable pulley?
- 2. In Fig. 71 what force at F is necessary to lift 100 lb. at W? What weight at W will just balance a downward pull at F of 230 lb.?
- 3. With arrangement as shown in Figs. 72 and 73, does the size of pulleys have anything to do with the force required to lift a given weight?
- **4.** In Fig. 72, how much force is necessary at F to raise a weight of 800 lb. at W?
- **5.** In Fig. 72, what weight at W will just balance a pull at F of 200 lb. if the lower block and hook together weigh 20 lb.?
- 6. In Fig. 73, what weight could a man weighing 150 lb. raise by hanging on the rope at the end F?

If the weight was raised 1 ft., through how many feet does the man move?

7. What force at F will a weight of 1000 lb. at W just balance

with the arrangement of Fig. 73, if the lower block and hook together weigh 26 lb.?

The Differential Pulley.—For lifting heavy weights by hand a differential pulley is often used.

As shown in Fig. 74, this consists of two pulleys A and B rigidly fastened together and rotating as one about a fixed axis C. An endless chain passes over both pulleys, and one of the loops of the chain passes under and supports the lower or movable block D. The other loop of the chain hangs freely and is the loop upon which

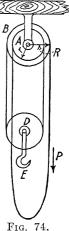
one pulls in raising a weight on the hook The rims of the pulleys are grooved and carry lugs to prevent the chain from slipping. If the radii of the large and small fixed pulleys are R and r respectively, "P" the pulling force exerted on the chain and "W" the weight lifted, the rule for the pulley is as follows, leaving out friction:

$$P \times R = W \times \frac{1}{2}(R - r)$$

$$\frac{W}{P} = \frac{R}{\frac{1}{2}(R - r)}$$

or

As used a differential pulley can actually lift only about 25 to 35 per cent. of the weight calculated for a given pull



from the formula stated above. This means therefore that to lift a given weight requires actually from three to four times the pull calculated by the formula which does not take account of friction.

Example.—What force is necessary to lift a weight of 800 lb. with a differential hoist having pulleys 10 in. and 81/2 in. in diam. and an efficiency of 25 per cent.?

Solution.—
$$P \times R = W \times \frac{1}{2}(R - r)$$
  
that is  $P = \frac{W \times \frac{1}{2}(R - r)}{R}$   
or  $P = \frac{800 \times \frac{1}{2}(5 - 4\frac{1}{4})}{5}$   
 $= \frac{800 \times \frac{1}{2} \times \frac{3}{4}}{5}$   
or  $\frac{800 \times \frac{3}{8}}{5}$  or 60 lb.

Since the efficiency is only 25 per cent. the actual pull required is  $\frac{60}{.25}$  or 240 lb.

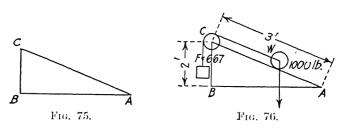
#### **PROBLEMS**

- 8. From the rule for a differential pulley find the force necessary to lift a tank weighing 280 lb. The diameters of the pulleys being 12 in. and 10 in. and the efficiency 30 per cent.
- 9. In a differential hoist with pulleys 11 in. and  $9\frac{1}{2}$  in. and an efficiency of 53 per cent., find what weight a 150-lb. man can raise by putting his own weight on the chain.

### CHAPTER VII

# THE INCLINED PLANE AND WEDGE. THE SCREW JACK

23. The Inclined Plane and Wedge.—A block whose sloping surface is less than 90° may be considered an "inclined plane." In Fig. 75 which shows such an inclined plane, AB is the "base," BC the "height" and AC the "face" or "sloping surface." With the use of the inclined plane a given resistance can be overcome with a smaller force than if the plane were not used. For ex-



ample, in Fig. 76, suppose we wished to raise a weight of 1000 lb. through the vertical distance BC = 2 ft. If this weight were raised vertically and without the use of the plane the force of 1000 lb. would have to be exerted through the distance BC. If, however, the inclined plane is used and the weight is moved over its face AC, a force of only  $\frac{2}{3}$  of 1000 lb. or 667 lb. is necessary, although this force is exerted through a distance AC which is greater than distance BC. With the

inclined plane, we, therefore, require a smaller force which must be exerted through a greater distance to do a certain amount of work.

Letting F represent the force required to raise a given weight on the inclined plane, and W the weight to be raised, we have the proportion:

$$\frac{F}{W} = \frac{\text{height of plane}}{\text{length of sloping surface of plane}}$$

or for Fig. 76

$$\frac{F}{W} = \frac{2}{3} \text{ and } \frac{F}{1000} = \frac{2}{3}$$

therefore  $F = \frac{2}{3} \times 1000$  or 667 lb.

Stated in words the rule for the "Inclined Plane" is:

The force required to raise a given weight over the plane is to the weight to be raised as the height of the plane is to the length of the sloping surface.

To find the force required, multiply the weight on the plane by the height of the plane and divide by the length of the face of the plane.

To find the weight which a given force will raise, multiply the force by the length of the face of the plane and divide by the height of the plane.

These rules do not include the effect of friction on the plane.

As shown in Fig. 77, a "wedge" consists of two inclined planes with their bases together. The force F used to drive the wedge acts parallel to the center line of the wedge.

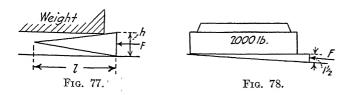
The principle of the wedge is the same as that for the inclined plane. In this case the wedge in being driven in is the same as if the weight being raised moved along a stationary inclined plane. Leaving out the effect of

friction on the sides of the wedge, the work done in driving the wedge equals the force exerted, F times l or Fl. The work done in raising the weight equals the weight lifted "W" times the distance lifted "h" or h. Then leaving out friction

$$Fl = Wh \text{ or } \frac{F}{W} = \frac{h}{l} \text{ or } F = W \times \frac{h}{l}$$

Therefore the force required is only  $\frac{h}{l}$  times the weight lifted.

Example.—In Fig. 78, a wedge 12 in. long has a taper



of  $1\frac{1}{2}$  in. per ft. and is used to raise a machine base weighing 1 ton. The force F required is found as follows:

$$F = \frac{W \times h}{l} = \frac{2000 \times 1.5}{12} = 250 \text{ lb.}$$

The "mechanical advantage" in using the wedge is therefore  $\frac{2000}{250}$  or 8. That is, the force required is only  $\frac{1}{8}$  of the weight lifted.

#### **PROBLEMS**

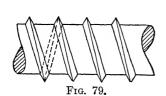
1. A weight of  $2\frac{1}{2}$  tons is to be raised by a wedge having a taper of 1 in. per ft. What force is required to drive the wedge and what is the mechanical advantage leaving out the effect of friction?

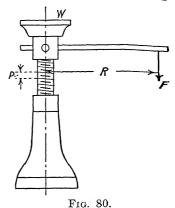
- 2. To wedge up a weight with a wedge tapering 2 in. per ft. requires a force of 500 lb. Neglecting friction, find the weight raised.
- 24. The Screw Jack.—One of the most common applications of the principle of the inclined plane is in the ordinary screw thread, which is nothing but an inclined plane wound around a cylinder.

If we cut out a wedge-shaped piece from thick paper and wind it around a cylinder we will see that the sloping

side forms a thread. The form of this thread as in Fig. 79 is called a "helix." All nut, bolt, jack, and screw threads are circular or spiral wedges.

Fig. 80 represents a screw





jack which is used to overcome a heavy pressure or raise a heavy weight at W by a much smaller force F applied at the handle. R represents the length of the handle and P the pitch of the screw, or the distance the screw advances in one complete turn.

Neglecting friction the following rule is used: The force F multiplied by the distance through which it moves in one complete turn is equal to the weight lifted times the distance through which it is lifted in the same time. In one complete turn the end of the handle describes a

circle of circumference  $2\pi R$ . This is the distance through which the force F is exerted.

Therefore from the rule above

$$F \times 2\pi R = W \times P$$
  
 $\pi = 3.14$  and  $F = \frac{W \times P}{2\pi R}$ 

Suppose in Fig. 80, R = 18 in.,  $P = \frac{1}{8}$  in. and the weight to be lifted is 50 tons or 100,000 lb., the force in lb. required at F is then found as follows:

$$F = \frac{W \times P}{2\pi R}$$

$$F = \frac{100,000 \times \frac{1}{8}}{2\pi 18} \text{ or }$$

$$F = \frac{100,000}{8} \times \frac{1}{2\pi 18}$$

Cancelling we get  $F = \frac{3125}{\pi 9} = \frac{3125}{28.3} = 110 \text{ lb. necessary.}$ 

This means then that neglecting friction 110 lb. at F will raise 100,000 lb. at W, but the weight lifted moves much slower than the force applied at F.

## **PROBLEMS**

- 3. What weight can be held by a force of 100 lb. on a plane which rises 4 ft. in every 5 ft. measured horizontally?
- **4.** The height of an inclined plane is 10 ft. and its length is 20 ft. What weight will a force of 25 lb. at F in Fig. 76 hold up on the plane?
- 5. What counterbalance would be necessary to hold an empty coal car weighing 500 lb. on an incline 20 ft. long and 8 ft. high?
- 6. A screw jack has a single thread 8 threads per in, and the handle is 2 ft. long. What force is necessary at F to raise a weight of 2500 lb., neglecting friction?

7. What effect has increasing the pitch P, on the force necessary to raise a given weight if the length R does not change?

8. It is desired to jack up a machine with a square base and weighing 5 tons by four jacks, one at each corner. If the jacks have 6 threads per in. and  $1\frac{1}{2}$ -ft. handles, and the weight is evenly distributed, what force is necessary at F?

9. What effect has increasing the length of handle on the force F necessary to raise a given weight if the same pitch is used?

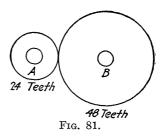
10. A force of 50 lb. at F raises a weight of 3768 lb. at W with  $R = 1\frac{1}{2}$  ft. What is the pitch of the screw?

## CHAPTER VIII

# GEARS, LATHE GEARING

25. Definitions.—A gear, or toothed wheel, when in operation, may be considered as a lever of the first class with the addition that it can be rotated continuously instead of rocking back and forth through a short distance. What we should learn here is the relation between the number of teeth, the diameter, and the speed of gears. Fig. 81 shows the ends of two shafts

A and B connected by 2 gears of 24 and 48 teeth respectively. Notice that the larger gear will make only one-half turn while the smaller makes a complete turn. That is, the ratio of speeds of the larger to the smaller is as 1 is to 2, or expressed as a proportion.



Speed of B: Speed of A = 1:2 and the ratio between the speeds and the number of teeth written as a proportion is

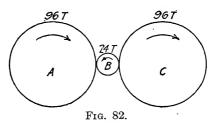
Speed of B: Speed of A = Teeth of A: Teeth of B. If in Fig. 81 gear A had 24 teeth and B 96 teeth, the smaller gear would make four turns while the larger makes 1, or,

Speed of A: Speed of B = 96:24, that is, the gear with the smaller number of teeth must turn the faster.

Of two gears running together that one is called the

driver which is nearer the source of power and the second gear which receives power from the driver is called the follower.

26. Gear Trains (Simple Gearing).—In the case of gear trains there may be several drivers and several followers. When the teeth on a gear turn in the same direction as the hands of a clock the motion is called right handed or "clockwise," and when in the opposite direction, left handed, or "counter-clockwise." In Fig. 82 suppose the three gears A, B, and C have 96, 24, and 96 teeth respectively.



When gear A turns once right handed, gear B turns 4 times left handed and gear C turns once right handed. Hence gear B does not change the speed of C from what it would have been if geared directly to A, but it changes its direction from left handed to right handed.

The ratio of speeds of the first and last gears in a train of simple gears is not changed by putting any number of gears between them.

27. Gear Trains (Compound Gearing).—Fig. 83 shows "compound gears" in which there are two gears on the middle shaft. Gears B and D rotate at the same speed since they are keyed to the same shaft. If the gears have a number of teeth as shown by the numbers in the

figure and gear A makes 100 r.p.m. right handed, gear B turns 400 r.p.m. left handed, also gear D turns 400 r.p.m. left handed and gear C turns 1200 r.p.m. right handed.

28. Lathe Gears for Screw Cutting (Simple Gearing).

—Modern lathes are equipped with a plate which gives the gears required for cutting different screw threads. To find the gears required in any case however it is merely necessary to understand the principle of a "simple" or a

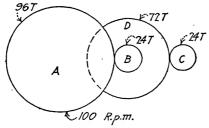


Fig. 83.

"compound" gear train depending on whether the lathe is simple or compound geared as explained in the following:

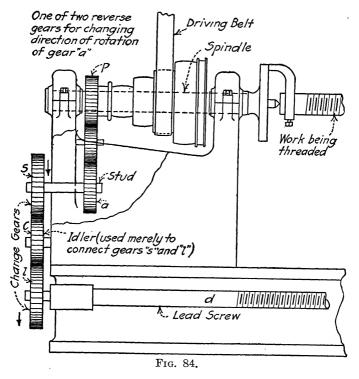
Fig. 84 shows a "simple geared" lathe with the names of important parts indicated. The lathe carriage carrying the cutting tool is moved by the lead screw which has usually 2, 4, 6, or 8 threads per in. If the lead screw has 4 threads per in., each complete revolution of this screw moves the carriage and hence the cutting tool ½ in. along the work being threaded, and 4 turns moves the tool 1 in. along the work. If now we wish to cut 12 threads per in., the spindle must revolve 12 times while the lead screw revolves 4 times. That is,

$$\frac{\text{turns of spindle}}{\text{turns of lead screw}} = \frac{12}{4} \text{ or as } \frac{3}{1}$$

The gears on the lead screw and on the spindle must therefore have a ratio of teeth of 3 to 1, or

$$\frac{\text{teeth on lead screw gear}}{\text{teeth on spindle gear}} = \frac{3}{1}$$

since the lead screw turns the slower.



If the gear a on the stud has the same number of teeth as gear p on the spindle, we have the rule

$$\frac{\text{teeth of lead screw gear}}{\text{teeth of stud gear}} = \frac{3}{1}$$

The idler gear (c) is used to connect gears S and l, and its number of teeth does not influence the speed ratio of the spindle and lead screw. The same is true of the gear connecting the spindle with the stud.

The following is the rule: The number of threads to be cut per in. equals the product of the number of threads per in. in the lead screw with the number of teeth in each driven gear and divided by the product of the number of teeth in each driving gear. In the figure shown, if the lead screw has 4 threads per in., gear p on the spindle 24 teeth, gear a on the stud 48 teeth, gear a 36 teeth, and gear a 72 teeth, the threads cut per in. equal  $\frac{4 \times 48 \times 72}{24 \times 36}$  or 16.

Using the letters on the gears to indicate their number of teeth we have the rule, if n = the number of threads cut

per in. and d =the number of threads per in. of lead screw  $n = \frac{d \times a \times l}{p \times s}$ 

For another example:

if d= threads per in. of lead screw =2 a= teeth in gear on stud =48 p= teeth in gear on spindle =24 s= teeth in outside gear on stud =24 l= teeth in lead screw gear =72Then  $n=\frac{2\times48\times72}{24\times24}=12$  threads per in.

Suppose on the other hand we wished to find gears "s" and "l" to cut 8 threads per in. with a lead screw

having 6 threads per in. we have from the rule,

$$n = \frac{d \times a \times l}{p \times s}, \text{ and } \frac{l}{s} = \frac{n \times p}{d \times a}, \text{ from which}$$
$$\frac{l}{s} = \frac{8 \times p}{6 \times a} = \frac{8}{6} \times \frac{p}{a}$$

to find the ratio  $\frac{l}{s}$  we must choose gears p and a. If we take p as 36 teeth and a 24 teeth, we have

$$\frac{l}{s} = \frac{8}{6} \times \frac{36}{24} = \frac{2}{1}$$

that is gears l and s must have a ratio of 2:1, hence we can put a 48-tooth gear on the lead screw and a 24-tooth gear as the outside gear on the stud and 8 threads per in. will be cut.

29. Lathe Gears for Screw Cutting (Compound Gearing).—In the compound geared lathe, as shown in Fig. 85, there are two change gears of different sizes on the spindle between the gear "s" on the stud and the gear "l" on the lead screw. The rule to be used in this case is as follows:

The threads to be cut per in. equal the product of the threads per in. of the lead screw and the teeth in each of the driven gears, divided by the product of the number of teeth in each of the driving gears. Representing the number of teeth of the gears by the letters on the gears we have, if "n" is the threads cut per in. and "d" the number of threads per in. of the lead screw.

$$n = \frac{d \times a \times l \times e}{p \times s \times f}$$

For example, suppose the lead screw has 5 threads per in. and the gears have a number of teeth as follows: a = 28, p = 28, l = 72, f = 36, s = 24, e = 24, the number of threads cut per in. according to the rule becomes

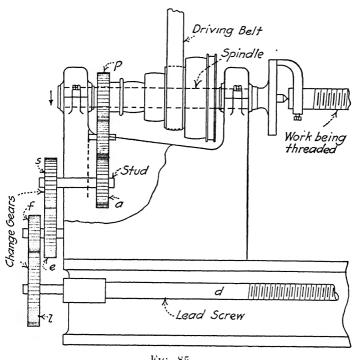


Fig. 85.

$$n = \frac{d \times a \times l \times e}{p \times s \times f} = \frac{5 \times 28 \times 72 \times 24}{28 \times 24 \times 36}, \text{ or } 10.$$

Working the other way round, if we wish to find the gears to use to cut a given thread we proceed as follows:

If d = 5 and the threads cut per in. are to be 14, we have

$$14 = \frac{5 \times a \times l \times e}{p \times s \times f}, \text{ from this}$$

$$\frac{14}{5} = \frac{a \times l \times e}{p \times s \times f}$$

The ratio of the gears must therefore be as 14 is to 5. Suppose by trial we take e = 28 and s = 30 teeth, we then have

$$\frac{14}{5} = \frac{a \times l \times 28}{p \times 30 \times f} \text{ or } \frac{14 \times 30}{5 \times 28} = \frac{a \times l}{p \times f}$$

that is  $\frac{a \times l}{p \times f} = \frac{3}{1}$ , the remaining ratio.

To provide this ratio we may take f = 24, l = 72, and a and p even gears as, for example, 36 teeth each. We then have

$$\frac{a \times l}{n \times f} = \frac{36 \times 72}{36 \times 24}$$
 or  $\frac{3}{1}$ 

# PROBLEMS

- 1. If 8 threads are to be cut per in. and the lead screw has 6 threads per in., what must be the ratio of revolutions for the spindle and lead screw?
- 2. In a simple geared lathe calculate the size of change gears to use to cut 14 threads per in. if the lead screw has 5 threads per in.
- 3. In a lather the ratio of revolutions of lead screw and spindle is  $\frac{1}{3}$ . If the pitch of the thread on the lead screw is  $\frac{1}{4}$  in., how many threads are cut for 16 revolutions of the lead screw?
- 4. In a compound geared lathe calculate the size of change gears to use to cut 18 threads per in. if the lead screw has 4 threads per in.

In working out the following problems think first of the relative sizes of the gears, which is the driver and which the follower, and remember that of two gears the larger must turn slower than the smaller. The diameters of gears are proportional directly to their number of teeth and inversely proportional to their speeds.

That is, if two gears A and B have pitch circles of diameters 12 and 24 in., respectively, and gear A has 48 teeth, gear B must have 96 teeth and if the speed of A is 100 revolutions per minute (r.p.m.) the speed of B is 50 r.p.m.

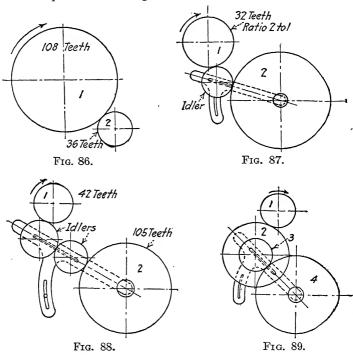
#### **PROBLEMS**

- **5.** If two gears A and B have 24 and 72 teeth respectively, and gear A turns right handed, in which direction will gear B turn? While gear A turns three times, how many times will B turn?
- **6.** If two shafts are to be connected by gears of 48 and 36 teeth, and the faster makes 900 r.p.m. what speed does the other make?
- 7. Two gears have one 120 and the other 90 teeth. What is the ratio of the diameters of their pitch circles and of their speeds?
- 8. Two gears have pitch circles 76 in. and 19 in. in diam. What is the ratio of their speeds?
- 9. It is desired to run a shaft at 200 r.p.m. from another running at 500 r.p.m., what gear must be used on the second shaft if the gear on the first shaft has 150 teeth?
- 10. If 7 gears work in a train in what direction will the last one turn if the first turns left handed? How could you change the direction of rotation of the last gear?
- 11. A train of gears is made up of 4 gears with the number of teeth as follows: 72, 48, 32, and 24. If the first gear makes 20 r.p.m. right handed, in what direction and at what speed will the last gear move?
- 12. A train of gears is made up of 5 gears with the number of teeth as follows: 48, 36, 24, 60, and 48. If the first gear makes 30 r.p.m. left handed, in what direction and at what speed will the last gear move?
- 13. A main shaft runs at 150 r.p.m., right handed, and carries a 96-tooth gear which drives a 24-tooth gear on a second shaft. A 72-tooth gear on the second shaft drives a 24-tooth gear on a third shaft. Find the speed of each shaft and make a sketch showing the arrangement of gears and directions of rotation produced.

14. A simple gear train has four gears with the number of teeth as follows: 96, 24, 48, and 60. If the last gear is on a shaft making 500 r.p.m. and turns right handed, find the speed and direction of rotation of the first gear.

15. According to Fig. 86, in what direction will gear 2 turn?

Find the speed ratio of the gears.



16. What is the direction of rotation of gear 2 according to Fig. 87? Find the number of teeth in gear 2.

17. Fig. 88 shows a train of simple gears. In what direction will gear 2 turn? Find the speed ratio of gears 1 and 2.

18. Fig. 89 shows a train of compound gears. In what direction will gear 4 turn.

## CHAPTER IX

#### BELTS AND PULLEYS. EFFICIENCY OF MACHINES

30. Belts and Pulleys.—Belts and pulleys are an important part of shop equipment and it is necessary to understand how to calculate speeds and diameters of pulleys and also lengths of belting.

Pulleys are nothing but gears without teeth and instead of running together directly they are made to drive one another by cords, ropes, cables, or belting of some kind.

As with gears, the speeds of pulleys are inversely proportional to their diameters. That is, if, as in Fig. 90, we have two pulleys A and Bwhich are 24 in, and 12 in, in

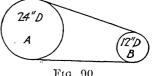


Fig. 90.

diam. respectively, the larger pulley travels the slower and the speed of A is to the speed of B as 12 is to 24.

This difference between gears running together directly and pulleys driven through belting should be fully understood. A belt "creeps" and "slips" and does not run the driver quite as fast as would be figured out from the diameters of the pulleys, while gears cannot slip and the practical speed and calculated speeds are the same.

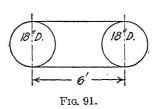
When two pulleys are driven through a belt if there is no slipping of the belt, the rim speed of both pulleys is

or

the same and in Fig. 90 when pulley A makes 1 turn the belt passes over each pulley a distance equal to the circumference of pulley A or  $2\pi 12$  or  $24\pi$  in. and if the rim of pulley B has passed through  $24\pi$  in. for 1 turn of pulley A and its diameter is 12 in. it has made a number of turns in this time equal to  $\frac{24\pi}{12\pi}$  or 2. The following simple rule is convenient for calculating speeds of pulleys.

mple rule is convenient for calculating speeds of pulleys.

The diameter of the driving pulley multiplied by its



speed is equal to the diameter of the driven pulley multiplied by its speed. In calculating the length of belt to connect two pulleys if the pulleys are the same size and the belting is not crossed the length of belt required is equal to twice

the distance between the centers of the pulleys plus the circumference of one of the pulleys. In Fig. 91 the length of belt needed equals

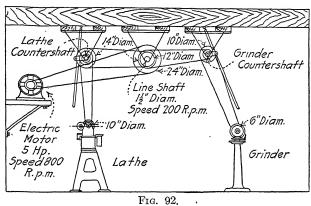
$$2 \times 6 + \pi \times \frac{18}{12}$$
  
12 + 4.7 or 16.7 ft.

If the diameters of the pulleys are not the same, but do not differ greatly and the distance between their centers is large compared with the pulley diameters, the length of belt is found very nearly by adding to twice the distance between the pulley centers half the circumference of each of the pulleys.

#### PROBLEMS

1. It is desired to run a countershaft at 200 r.p.m. from a line shaft running at 800 r.p.m. If the pulley on the line shaft is 18 in. in diam., what size pulley is required on the countershaft?

- 2. What length of belt is required to connect two 24-in. pulleys whose centers are 6 ft. 4 in. apart?
- **3.** A belt runs from a 36-in. pulley on a line shaft to a second shaft having a 14-in. pulley. If the line shaft makes 650 r.p.m. what is the speed of the second shaft?
- 4. Find the length of belt required to connect a 24-in. and a 36-in. pulley whose centers are 8 ft. apart.
- 5. A main shaft running at 150 r.p.m. with an 18-in. pulley drives a jack shaft through a 12-in. pulley. A second pulley of 24 in. diam. on the jack shaft drives a dynamo through a 12-



in. pulley. Find the speed of the dynamo and make a sketch showing the arrangement of pulleys, and direction of rotation of shafts.

6. Calculate from the following rule the length of belting required to connect two 24-in. pulleys 7 ft. apart if the belt is crossed?

$$l = 2\sqrt{D^2 + S^2 + \pi D}$$

When D = diam. of pulleys

S = distance between centers

7. A main shaft running at 200 r.p.m. with a 12-in. pulley drives a second shaft through a 24-in. pulley. The slip of the belt is 2 per cent. of the driving speed. On the second shaft is a second pulley 36 in. in diam. which is connected through a belt to an 18-in. pulley on an electric generator. The belt on the generator

slips 3 per cent. of the driving speed of the second shaft. What is the speed of the generator?

8. Fig. 92 shows the elevation of a layout for a small machine shop, with the equipment as indicated. Working from this sketch, find

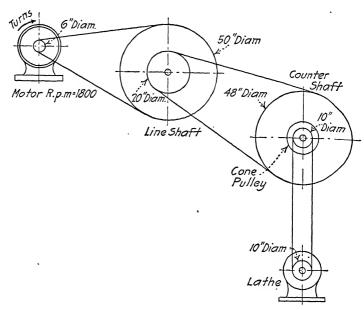


Fig. 93.

- 1. The diameter of pulley on motor shaft.
- 2. The speed in r.p.m. of lathe countershaft.
- 3. The speed of lathe. Pulley on lathe countershaft is 12 in . in diameter.
- 4. The speed of grinder countershaft. Pulley on grinder countershaft is 8 in. in diameter.
- 5. The speed of grinder.
- 6. The weight in pounds of line shaft. Length 10 ft., diam. 2½ in. Material, steel weighing .28 lb. per cu. in.

- 9. With the belts as arranged in Fig. 93, show the direction of turning of the lathe spindle, with the direction of rotation shown for motor. How could the direction of turning of the lathe be reversed? With data given on sketch what is the speed in r.p.m. of the lathe spindle when belted as shown? Why is it considered better practice to have the tight side of the belt on the bottom of the pulleys?
- 31. Efficiency of Machines.—In working out the problems on levers, pulleys, inclined plane and so forth we have not taken account of friction or other sources of loss. In other words we have supposed them to be perfect. As actually used their operation is not as satisfactory as our answers show, and as an indicator of the performance of a machine we often find its "efficiency."

Efficiency means  $\frac{\text{the output of a machine}}{\text{input to the machine}}$  or better  $\text{efficiency} = \frac{\text{output}}{\text{input}}$ .

The efficiency is a number which shows how much of the energy the machine takes, it puts out again as useful work.

Suppose a screw jack has an efficiency of 20 per cent. or .20. This means that it delivers in useful work 20 per cent. of all the energy or work put into it.

If a machine had an efficiency of 100 per cent. or 1 it would mean that efficiency =  $1 = \frac{\text{output}}{\text{input}}$  therefore input  $\times$  1 = output, or output = input. But no machine has an efficiency of 100 per cent. and therefore the output can never equal the input since there are always some losses in the machine itself.

Suppose a 50-h.p. motor has an efficiency of 85 per

or

cent. when delivering its rated (50) horsepower.  $T_0$  find its input we have

Efficiency = 
$$\frac{\text{output}}{\text{input}}$$
, therefore input =  $\frac{\text{output}}{\text{efficiency}}$  or input =  $\frac{50}{.85}$  or 58.8 h.p.

If a steam engine delivers 50 h.p. and its input is 70 h.p., its efficiency is found as follows:

Eff. = 
$$\frac{\text{output}}{\text{input}} = \frac{50}{70} = .714$$
  
.714 × 100 = 71.4 per cent.

It is therefore to be remembered that in working out a problem on machines we always have to take account of losses that are due to friction in the machines themselves in order to get the correct results.

### **PROBLEMS**

- 10. An electric generator delivers 48 h.p. and requires 60 h.p. to drive it. What is its efficiency?
- 11. A screw jack lifts a 20-ton weight 2 ft. thereby doing  $20 \times 2000 \times 2$  or 80,000 ft.-lb. of work, and in doing this work requires 334,000 ft.-lb. of work. Find its efficiency.
- 12. An engine is rated at 100 h.p. and has an efficiency at this load of 75 per cent. What is its input?
- 13. An electric motor takes 20 h.p. to do a certain amount of work. If its efficiency is 88 per cent., find its horsepower output.

## CHAPTER X

### MOTION

**32.** Definitions.—Motion is a change of position and may be of a number of kinds.

In order to set objects in motion forces or pressures are necessary. If a steady force acts on an object moving it, the object increases in motion until the resistance with which the object meets is equal to the force causing the motion and the object then travels over equal distances in equal times or as we say, has a constant speed. The word constant means unchanging, and speed, or what is the same thing, velocity, means the rate of motion of an object. If a train going at a steady rate covers fifteen miles in an hour its speed or velocity is 15 miles per hour (abbreviated m.p.h.). When an object is moving at a constant speed, if the driving force is removed, the speed decreases until the object is brought to rest by resisting forces such as friction, etc. When an object increases in speed it is said to "accelerate," and when it decreases in speed it is said to "retard."

Any object at rest will remain at rest for all time and an object in motion will always move at a constant speed in a straight line unless it is acted upon by a force or pressure which deflects it or changes its rate of motion, and how much an object will change in speed or direction or both depends on how large the force is acting on it.

When an object is free to fall the force of attraction of

the earth for it or the force of gravity causes the object to gain in speed 32.2 ft. per sec. each second of its fall.

33. Composition of Motions.—Motions, like forces, may be conveniently represented by straight lines. The length of the line representing to scale the rate of the motion as 10 ft. per sec. or 40 m.p.h. and the direction of the line, the direction of the motion. In Fig. 94 an object at O is represented as having two successive motions of 10 and 15 miles per hour, these motions being 60° apart. The effect of the two motions is the same



Fig. 94.

as one motion of 22 m.p.h. in the direction OC. This occurs, for example, if from a train traveling in the direction OB at 10 m.p.h., an object is thrown off in the direction OD at a speed of 15 m.p.h., the object itself is affected by both motions at the same time and travels in the direction

OC at the rate of 22 m.p.h. If two trains are traveling in the *same* direction, one at 40 m.p.h. and the other at 35 m.p.h. the *relative* speed of the trains is only 5 m.p.h.

If an engine hauls a freight train at the rate of 20 m.p.h. and a brakeman runs along the train in the direction of motion at a speed of 5 m.p.h. he is traveling at the rate of 20 + 5 or 25 m.p.h. with respect to the earth or track.

The following rules are convenient in determining speeds.

Velocity or speed =  $\frac{\text{distance}}{\text{time}}$ and  $\text{distance} = \text{speed} \times \text{time}$ 

Acceleration or rate of change of speed  $=\frac{\text{speed}}{\text{time}}$ 

If a train starts from rest and increases uniformly in speed to 24 m.p.h. in 3 min. it has changed in speed or "accelerated"  $\frac{24}{3}$  or 8 m.p.h. each minute. If this constant rate of increase of speed were kept up for 5 min. by a locomotive starting from rest, its speed at the end of the 5 min. would be  $8\times 5$  or 40 m.p.h.

It has already been stated that the earth attracting a freely falling object increases its speed about 32.2 ft. per sec. for each second of its fall.

#### **PROBLEMS**

- 1. A locomotive went from Altoona, Pa. to Harrisburg, Pa., a distance of 131 miles in 112 min. What was its average speed in miles per hr., and in ft. per sec.?
- 2. The same locomotive mentioned in Prob. 1 went from Altoona to Philadelphia, a distance of 236 miles in 3 hr. and 19 min. Making allowance for a 3-min. stop at Harrisburg, find the average speed made in miles per hr. and in ft. per sec.
- **3.** A passenger locomotive leaving a station attains a speed of 40 m.p.h. in 4 min. 30 sec. What is the acceleration or rate of change of speed?
- 4. An engineer has to make, in 3 hr., a 120-mile run with four 3-min. stops. What average rate of speed does he have to maintain in order to complete the run on time?
- 5. A passenger train making an average speed of 38 m.p.h. has to make a run of 85 miles. How much sooner will it complete the run than a freight which starts at the same time and makes an average of 25 m.p.h.?
- 6. The "Pennsylvania Limited" traveling at 60 m.p.h. starts from a station an hour after an accommodation train. The accommodation is delayed ½ hr. and is overtaken by the "Limited in 2 hr. What is the rate of speed of the accommodation train and how far had it traveled when overtaken by the "Limited?"
- 7. If the 82-in. drivers on a passenger locomotive make 200 revolutions per minute (r.p.m.) what is its speed in miles per hr.? What would be its speed allowing for a 2 per cent. slip?

- 8. A passenger locomotive with 80-in. drivers making 180 r.p.m. and a freight engine with 60-in. drivers making 150 r.p.m. are traveling in the same direction. What is the relative speed of the locomotives? If the locomotives were traveling in opposite directions what would be their relative speeds?
- 9. An engineer makes an average of 42 m.p.h. and runs for 4 hr. and 20 min. What distance does he cover?
- 10. If an object falls freely from rest for 3 sec., what is its speed in ft. per sec. and in miles per hr. at the end of that time?
- 11. The drivers on a locomotive are 18.85 ft. in circumference and make 250 turns a min. What is the speed of the locomotive in miles per hr.?
- 12. Two trains start from a station at the same time and travel over level country, the first North at the average rate of 40 m.p.h. and the second East at the average rate of 30 m.p.h. How far apart are the trains at the end of 5 hr.?
- 13. A train makes a total distance of 236 miles, covering one-third of this distance at the rate of 45 m.p.h. and the remaining distance at the rate of 40 m.p.h. In what time does it make the trip if its total stops take 10 min.?
- 14. A touring car starts from a garage and reaches a speed of 48 m.p.h. in 4 min. and 20 sec. What is its acceleration in miles per hr. per min., and in miles per hr. per sec.?
- 15. Find the speed in ft. per sec. and in miles per hr. of a freely falling object at the end of the fourth second of its fall.
- 16. In a trolley car the gear on the driving axle and the gear on the motor axle have a ratio of teeth of 4:1. The driving wheel has a diameter of 32 in. Find the speed of the trolley car in miles per hr., when the motor is making 1000 r.p.m.
- 17. Two trains start from a station at the same time, one going East at 40 m.p.h. for 5 hr. and the other North at 20 m.p.h. for 2 hr. and then East at 10 m.p.h. for 3 hr. Find the distance apart of the trains at the end of the 5 hr.

### CHAPTER XI

### CUTTING SPEEDS. SPEEDS OF LATHES

34. Definitions and Problems.—The calculation of the circumference and diameter of circles becomes necessary in calculating the cutting speed of work being turned off in a lathe or in calculating the rim speed of fly-wheels, grindstones, etc.

Machines, such as lathes, milling machines, boring mills, planers, etc., are provided with means for changing the speed according to the work turned out. Every mechanic should know how to calculate the proper size of pulleys or gears for any particular job. When a piece of metal of circular section is being turned down in a lathe the cutting tool for each revolution of the work travels a distance (with respect to the metal being turned down) equal to the circumference of the work.

For example, a rod 2 in. in diam. passes a distance of  $3.142 \times 2$  or 6.284 in. under the cutting tool for each revolution of the rod. If the rod rotated 60 times per min. the cutting speed would be 60 times 6.28 or 376.8

in. per min. or  $\frac{376.8}{12}$  or 31.4 ft. per min.

If S is the surface speed in ft. per min.,

N the revolutions per min. of the work,

C the circumference of the work in in.,

the rule is 
$$S = \frac{C \times N}{12}$$

For example, a 2-in. diam. brass rod when turned at the rate of 160 r.p.m. has a surface speed of

$$S = \frac{2 \times 3.142 \times 160}{12}$$
 or 83.8 ft. per min.

The rim speed of a fly-wheel 72 in. in diam. when running at 200 r.p.m. may be found by the same rule as follows:

$$S = \frac{72 \times 3.142 \times 200}{12}$$
 or 3770 ft. per min.

Working the other way round we can find the revolutions per min. (N) at which a wheel must rotate to have a given surface or rim speed.

If a fly-wheel 48 in. in diam. is to have a rim speed of a mile a min. or 5280 ft. per min. its revolutions per min. are found from the rule

$$N = \frac{12 \times S}{C}$$
, that is Revolutions per min.  $= \frac{12 \times \text{speed in ft. per min.}}{\text{circumference of wheel in in.}}$  herefore  $N = \frac{12 \times 5280}{48 \times 3.142} = 420 \text{ r.p.m.}$ 

Therefore

There are average values given for cutting speeds of different metals as follows:

Cutting Speeds in Ft. per	Min.
Brass	80
Machinery steel	30
Tool Steel	20
Cast Iron	40

With high-speed steel cutting tools about double the above speeds can be used. No exact figures can be given for the best cutting speeds of the different metals.

proper speed to use depends on the size of the work, the power and rigidity of the lathe, the kind of tool used and the nature of the work being turned out.

For the surface speed of grindstones the following figures may be taken

For carpenters' tools, 600 ft. per min. For machinists' tools, 1000 ft. per min.

When edge tools are ground care must be taken to avoid too high a speed to prevent heating the edges of the tools and getting rough surfaces. For rapid grinding the following speeds are used:

With Huron stones, 3500 ft. per min. With Ohio stones, 2500 ft. per min.

Some polishing wheels are run as high as 8000 ft. per min. Emery wheels are run usually at a surface speed of about a mile a min. or 5280 ft. per min.

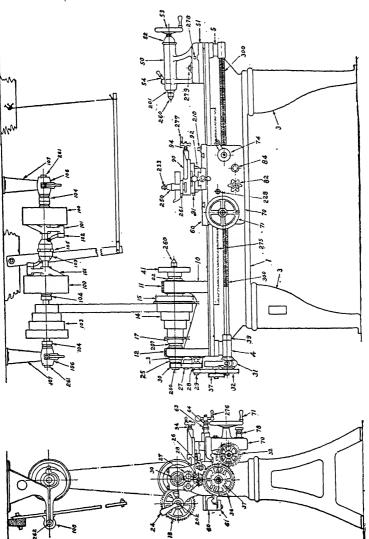
To find how long it will take the cutting tool on a lathe to travel a given distance along the work with a given feed we proceed as follows:

Suppose a lathe tool has a feed of  $\frac{1}{8}$  in. per rev., and we wish to find how long it will take to turn a length of 18 in. on an 8-in. diam. piece with a cutting speed of 60 ft. per min. The circumference of the piece is  $8 \times 3.142$  or 25.136 in. This is equal to  $\frac{25.136}{12}$  or 2.094 ft. With a cut-

ting speed of 60 ft. per min. this requires  $\frac{60}{2.094}$  or 28.6 r.p.m.

For each revolution with a feed of  $\frac{1}{8}$  in, the tool travels along the work  $\frac{1}{8}$  of an in, or  $28.6 \times \frac{1}{8} = 3.57$  in, per min. To turn a length of 18 in, would therefore require

 $\frac{18}{3.57}$  or 5.04 min. or a little over 5 min.



Cut of Lathe showing principul paris numbered,

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## NUMBER AND NAME OF LATHE PARTS ON DRAWING

NO.

1. Bed.

3. Power Legs.

4. Lead Screw Bracket F.

5. Lead Screw Bracket R.

10. Head Stock.

11. Head Stock Cap, Large.

12. Head Stock Cap, Small.

13. Head Stock Clamp Plate.

14. Spindle Cone.

15. Bull Gear.

16. Bull Gear Clamp.

17. Conc Pinion.

18. Quill Gear.

Quill Sleeve.

20. Quill Sleeve Pinion.

21. Ecc. Shaft Bushing.

22. Bronze Box, Large.

23. Bronze Box. Small.

24. Back Gear Lever.

25. Spindle Take up Nut.

26. Reverse Bracket.

27. Reverse Twin Gears (2).

28. Reverse Gear.

29. Stud Gear.

30. Spindle Reverse Gear.

31. Change Gear Bracket.

32. Change Gears.

33. Change Gear Idler.

34. Bushing for Change Gear Idler.

Change Gear Collar on L. S.

36. Compound Idler Gear 1 to 2 Large.

37. Compound Idler Gear 1 to 2

38. Bushing for Compound Idler

Gear, I to 2.

39. Thrust Collar on Lead Screw.

40. Large Face Plate.

41. Small Face Plate.

42. Turning Gear.

50. Tail Stock Top.

51. Tail Stock Base.

52. Tail Stock Nut.

53 Ta' Stoc Hand Wheel

NO.

56. Tail Stock Clamp Plate.

60. Saddle.

61. Saddle Gib.

62. Saddle Lock.

63. Cross Feed Bushing.

64. Cross Feed Gra. Collar.

65. Cross Feed Nut.

66. Plain Rest.

67. Thread Cutting Stop.

70. Apron.

71. Apron Hand Wheel.

72. Lead Screw Half Nut.

73. Lead Screw Half Nut Gib (2).

74. Nut Cam.

75. Nut Cam Washer.

76. Rack Pinion Gear.

77. Auto. Apron Worm Wheel.

78. Auto. Apron Clutch Sleeve Bushing.

79. Auto. Apron Worm Bracket.

80. Auto. Apron Clutch Sleeve.

81. Auto. Apron Clutch.

82. Auto. Apron C. F. Star Knob. 83. Auto. Apron C. F. Lever.

84. Auto. Apron C. F. Lever Knob.

85. Auto. Apron C. F. Gear.

86. Auto. Apron Idler Gear.

87. Auto. Apron Idler Gear Pinion.

90. Compound Rest Top.

91. Compound Rest Swivel.

92. Compound Rest Bottom.

93. Compound Rest End Cap.

94. Compound Rest Bushing.

95. Clippound Rest Nut.

96. Compound Rest Chip Guard.

#### Countershaft

100. C. S. Friction Pulleys (2).

101. C. S. Friction Spiders (2).

102. C. S. Friction Fingers (2).

103. C. S. Cone.

104. C. S. Collars (4).

105 C S Vok Lever

NO.

108. C. S. Shipper Nut.

109. C. S. Yoke Cone.

200. Head Stock Spindle.

201. Tail Stock Spindle.

202. Back Gear Eccentric Shaft.

203. Apron Worm.

204. Apron Rack Pinion.

205. Spindle Sleeve.

207. Spindle Thrust Collar.

208. Apron Worm Collar.

209. Tool Post Block.

210. Carriage Lock Collar Screw.

211. Compound Rest Swivel Bolts.

212. C. G. Bracket Collar Screw.

213. Reverse Collar Screw.

214. Bull Gear Clamp Collar Screw.

215. Apron Clutch Sleeve Hex. Nut.

216. Compound Rest Swivel Stud.

217. Steady Rest Lock Bolt.

218. Auto Cross Feed Lever Stud.

219. Reverse Steel Washer ½" hole.

220. Apron Clutch Sleeve Pinion.

221. Compound Rest Bottom Gib.

222. Plain Rest Gib.

223. Auto Apron Clutch Screw.

224. Cross Feed Screw.

225. Apron Hand Wheel Pinion.

226. Tail Stock Screw.

227. Reverse Shaft or Stud.

228. Apron Rack Pinion Stud.

229. Reverse Shoulder Screws (2).

NO.

230. Compound Rest Screw.

231. Auto Cross Feed Stud.

232. Apron Half Nut Stud (2).

233. Tool Post Screw.

234. Apron Idler Gear Stud.

235. Cam Cap Screw.

238. Apron Worm Washer.

239. Comp. Rest Steel Wedge.

240. Gap Bridge Pins (2).

241. Reverse Stud Collar 34" hole.

242. Change Gear Spindle Knob.

250. Tool Post.

251. Tool Post Ring.

252. Tool Post Wedge.

253. Tool Post Wrench.

254. Compound Rest Wrench.

257. Compound Rest Top Gib.

258. Comp. Rest C. P. Headless Set Screws.

260. Centers (2).

261. C. S. Shaft.

262. C. S. Shipper Rod.

263. C S. Expansion Wedges.

275. Rack.

276. Cross Feed Ball Crank.

277. Compound Rest Handle.

278. Tail Stock Set Over Screws (2).

279. Tail Stock Clamping Bolt, Nut and Washer.

300. Lead Screw.

## **PROBLEMS**

1. A steel bar 3 in. in diam. is to be turned in a lathe. If the cutting speed is to be 30 ft. per min. at how many revolutions per min. should the bar turn?

2. A cast-iron pulley 18 in. in diam. is to have its rim face turned down in a lathe. With a cutting speed of 40 ft. per min. what should be the revolutions per min. of the pulley?

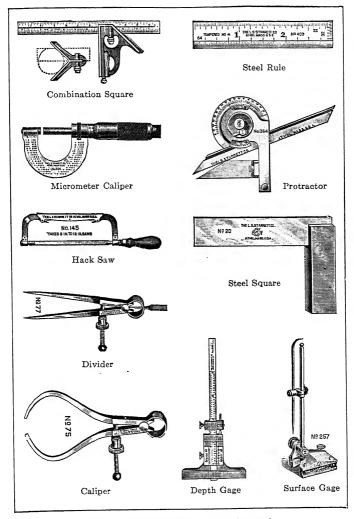


Fig. 96 —Cut showing machinist tools.

3. An emery wheel 14 in. in diam. runs at 1380 revolutions per minute (r.p.m.). Find its surface speed in ft. per min.

4. What is the speed of a belt running on a 3-ft. pulley which

makes 300 r.p.m.?

5. At how many revolutions per min. should we run a 2-in. diam. brass rod in a lathe for a cutting speed of 80 ft. per min.?

- 6. Find the revolutions per min. for a  $1\frac{1}{8}$ -in high-speed drill to give a cutting speed at the outer edge of 80 ft. per min. If the feed is .015 in. per rev., how long will it take to drill through  $1\frac{1}{2}$  in. of metal?
- 7. Find the revolutions per min. of an 18-in. polishing wheel whose surface speed is 8000 ft. per min.
- 8. At how many revolutions per min. should a 30-in. grindstone be run to obtain the proper surface speed for grinding machinists' tools?
- 9. A piece of work 6 in. in diam. is being turned down at a cutting speed of 60 ft. per min. (f.p.m.). If the cutting tool feeds along the work ½ in. for each revolution, how long will it take to turn a length of 8 in.?
- 10. A steel crank shaft is to be finished to a diameter of 8 in. With a cutting speed of 40 f.p.m. what should be its speed in revolutions per min.?

## CHAPTER XII

## VOLUME AND PRESSURE OF GASES

35. Definitions and Problems.—A very important law has been discovered in regard to the relation of the volume, temperature and pressure of gases. Steam follows this law very closely. It has been found that if the temperature of a gas does not change, its volume is practically "inversely proportional" to its pressure, or its volume times its pressure is a constant quantity. This means that if we have a cylinder, as in Fig. 97, which contains

10 cu. ft. of air with the piston in position A and the gauge reads 10 lb., making an absolute pressure of 10 + atmospheric pressure (that is, 10 + 14.7 or 24.7 lb.), and we move the piston to the posi-

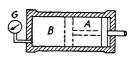


Fig. 97.

tion B so that the volume is only 5 cu. ft. or one-half of its former value, the absolute pressure becomes  $2 \times 24.7$  lb. or 49.4 lb. or twice its former value and the gauge reads 49.4-14.7 or 34.7 lb. since the gauge is made to read pressures above that of the atmosphere. That is, making the gas occupy half the volume has doubled its pressure. If, on the other hand, we increase the volume of the gas, we lower its pressure. This is exactly what happens in a steam engine cylinder into which steam enters at perhaps 200-lb. pressure, then expands, does work

in moving the piston, becomes reduced to a comparatively low pressure and exhausts into the air.

The principle of expansion and compression of gases is made extensive use of in all steam and gas engines and in air-braking devices. The rule is: If the temperature of a gas does not change, the product of its volume and pressure remain practically the same throughout a considerable range of pressure.

#### PROBLEMS

1. If the air in a cylinder containing 5 cu. ft. at atmospheric pressure is compressed to a volume of 1 cu. ft., what is its pressure?

2. A gas occupies 10 cu. ft. at an atmospheric pressure of 15 lb. per sq. in. and is afterward compressed to a volume of 2 cu. ft. What is its pressure?

3. During the movement of a piston in a steam cylinder the pressure drops from 90 lb. to 50 lb. If this change took place according to the law explained, what change of volume took place?

4. A 30-in. × 36-in. cylinder with a .2 cu. ft. clearance volume at each end takes in air at atmospheric pressure of 14.7 lb. What is the pressure of this air when the piston has completed one-half the return stroke and is compressing the free air drawn in on the forward stroke?

5. Twenty cubic feet of free air is compressed to a pressure of 60 lb. gauge. What volume does it then occupy?

6. The air cylinder of an air compressor is 20 in. × 28 in. and its clearance volume is .2 of a cu. ft. What is the pressure of the discharged air if the outlet valves lift at ¾ stroke?

7. If the admission of steam to a 24-in. × 26-in. cylinder at a pressure of 200 lb. per sq. in., gauge is cut off at 50 per cent. of the piston stroke (½ stroke) what is the terminal pressure, that is, the pressure at the end of the stroke?. (Give the answer in gauge pressure.) Clearance volume at each end of cylinder is .2 cu. ft.

8. What would be the terminal pressure if the cut-off occurs at 25 per cent. of the stroke (1/4 stroke) with the conditions of Prob. 7?

- 9. With a back pressure on the piston during the exhaust stroke of 3 lb. per sq. in. gauge, what would be the compression pressure (lb. per sq. in. gauge) at the end of the stroke when compression or exhaust closure occurs at .6 of the piston stroke, in a 24-in.  $\times$  26-in. cylinder? Clearance volume at end of cylinder is .2 cu. ft.
- 10. What would be the compression pressure (lb. per sq. in. gauge) if compression of the steam occurs at  $\frac{7}{8}$  stroke with other conditions the same as in Prob. 9?

### CHAPTER XIII

### WORK AND POWER.

36. Definitions and Rules on Work.—Work means the overcoming of resistance of any kind. If you drag an object along the floor you do work in overcoming the friction between the object and the floor. In lifting an object you do work against gravity which tends to pull the object toward the earth. Steam in a locomotive cylinder does work when it expands and moves the piston against the resisting forces. "Work" is measured in "footpounds," that is, the product of the resistance overcome in lb. and the distance in ft. through which it is overcome. If you lift a 10-lb. weight 5 ft. you do  $10 \times 5$  or 50 ft.-lb. of work against gravity.

If a train weighs 350 tons and the resistance offered to its motion is 12 lb. per ton, the work done in moving the train 1 mile at uniform speed is

$$350 \times 12 \times 5280 = 22,176,000 \text{ ft.-lb.}$$

If it takes an average pressure of 30 tons to punch a  $\frac{3}{6}$ -in. steel plate, the work done equals the resistance in lb., or  $30 \times 2000$  lb. times the thickness of the plate in ft.,  $\frac{3}{6}$  in. =  $\frac{1}{3}$ 2 ft.

Therefore work done = 
$$30 \times 2000 \times \frac{1}{32}$$
  
=  $1875 \text{ ft.-lb.}$ 

37. Power, Horsepower.—Power means the rate at which work is done and is expressed in foot-pounds per

min. or in foot-pounds per sec. When 550 ft.-lb. of work are done in a sec. or 33,000 ft.-lb. in a min. 1 h.p. is represented. A 10-horsepower (abbreviated h.p.), engine therefore is one which can do  $33,000 \times 10$  or 330,000 ft.-lb. of work in 1 min., or is one capable of overcoming a resistance of 330,000 lb. through a distance of 1 ft. in 1 min.

The rule for horsepower is:

Horsepower =  $\frac{\text{pounds} \times \text{feet}}{550 \times \text{seconds}}$ Horsepower =  $\frac{\text{pounds} \times \text{feet}}{33,000 \times \text{minutes}}$ 

or

In electrical terms 1 h.p. is equal to 746 watts. The horsepower required in any case depends as much on the *time* in which the work is to be done as on the *amount* of work to be done.

#### PROBLEMS

1. What is the horsepower of an engine that will raise 8000 lb. of water 200 ft. in 5 min.?

2. What is the horsepower of an automobile engine which can do 82,500,000 ft.-lb. of work in 5 min.?

3. What horsepower motors are required to raise a 222,000-lb. locomotive 4 ft. in 1 min.?

**4.** If a man in a day of 8 hr. raises 20,000 lb. of gravel through an average distance of 5 ft., what horsepower represents his working power?

5. A man weighing 150 lb. runs up stairs to a vertical height of 20 ft. in 2 min. What horsepower is he exerting to raise his own weight?

6. What is the useful horsepower of an engine which draws a train of 200 tons on a level at the uniform rate of 30 miles per hr. against a resistance of 16 lb. per ton weight?

7. A 7-h.p. engine is used to raise a weight of 24 tons. How high will it raise it in 1 min.? If it is desired to raise the weight in twice the time, what horsepower is required?

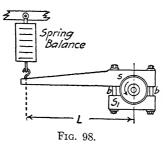
- 8. It is desired to raise a weight of 5 tons 200 ft. in 40 min. What horsepower is necessary to do the work?
- 9. What average horsepower is required merely to lift a 250-ton train up 5 miles of grade rising 1 ft. in every 528 ft., if the train is moving at the rate of 25 miles per hr.?
- 10. If the name plate on a motor gives the motor rating as 5 h.p., how many foot-pounds of work should the motor do per min. without overheating? Can the motor be worked above its rated horsepower?
- 11. From the rule that horsepower per cylinder  $=\frac{PLAN}{550}$  in which P is the average steam pressure in pounds per sq. in., L the length of stroke in ft., A the area of the piston in sq. in., and N the number of strokes per sec., find the horsepower developed on both sides of a locomotive whose cylinders are 20 in.  $\times$  28 in., working with an average steam pressure of 180 lb. per stroke and 150 strokes per min.?
- 12. A 12-ton casting is raised 20 ft. in 1 min. by a traveling crane. What horsepower does the motor on the crane exert?
- 13. From the rule for horsepower per cylinder, h.p. =  $\frac{PLAN}{33,000}$  in which P is the average pressure in pounds per sq. in. on the piston, L the length of stroke in feet, A the area of piston in square inches, and N the strokes per min. (or 2 times the r.p.m.) find the horsepower for both cylinders of the following locomotive. Cylinders 24 in.  $\times$  26 in. (24 in. diam. of piston and 26 in. stroke of piston), average steam pressure 160 lb. per sq. in., and running at 200 r.p.m.
- 14. From the rule in the foregoing problem calculate the total cylinder horsepower of a 4-cylinder Mallet locomotive.

Cylinders 27 in. × 28 in., average steam pressure on piston 130 lb. per sq. in., and r.p.m. 150.

15. Find the horsepower of a freight locomotive running at 10 m.p.h. with an average cylinder pressure of  $\frac{4}{5}$  boiler pressure. Drivers 62 in. in diam., cylinders 24 in.  $\times$  28 in., boiler pressure 205 lb. per sq. in.

The power delivered by an engine or a motor at its pulley, that is, the actual power available for doing work may be found for small machines by the Prony-Brake Method.

Fig. 98 shows such a brake consisting of shoes s and  $s_1$ , which are clamped to the machine pulley by bolts b and b, which may be regulated to increase or decrease the pressure of the shoes on the pulley. When the pulley rotates in the direction of the arrow, the entire brake and lever L tend to rotate in the same direction. This tendency is counterbalanced by the pull on the spring shown, from which we may read the pounds pull for different degrees of pressure of brake shoes on pulley.



The spring keeps the lever arm practically horizontal throughout the test. The power absorbed by the brake shoes "s" and "s<sub>1</sub>" equals the amount of work done by the revolving pulley on the machine. In using the brake we first take the reading of the

spring balance with the machine not running in order to account for the weight of the lever itself, then after bringing the machine up to speed and putting a "load" on it by means of bolts b and b, the balance reading is again taken, and the initial reading subtracted to get the net pull due to the rotation alone. If P is the pull in pounds on the balance, L the length of the lever as shown and r.p.m. the revolutions per min. of the pulley, the foot-pounds of work done per min. equals  $P \times 2\pi L \times r.p.m.$ , and the horsepower equals

$$\frac{P \times 2\pi L \times \text{r.p.m.}}{33,000}$$
.

When used in tests of this kind the machine pulley is made so that water may be run on the inside of the

rim to keep the pulley cool and absorb the heat produced by the transformation of the mechanical work done at the pulley.

The principle of the brake described above is the same as that of the power of a belt in which the effective belt pull is "P" pounds on a pulley of L radius and running at a speed represented by the symbol r.p.m. In this case the belt moves at a speed of  $2\pi L \times \text{r.p.m.}$  feet per min. and the horsepower is therefore  $\frac{P \times 2\pi L \times \text{r.p.m.}}{22,000}$  or

the same rule as derived above.

The principle of the brake may also be considered the same as that of a weight "P" being raised by a rope wound on a pulley of radius L, as in Fig. 99.

Example.—Find the horsepower delivered by a direct-current motor in a brake test in which the net effective pull at end of lever 2½ ft. long is 10 lb., with the pulley rotating at 900 r.p.m.

$$\begin{array}{c} -L \rightarrow \\ P \end{array}$$

Fig. 99.

Solution.—Horsepower =  $\frac{P \times 2\pi L \times \text{r.p.m.}}{33,000}$ or  $\frac{10 \times 2 \times 3.14 \times 2.5 \times 900}{33,000}$ 

which equals 4.282 or practically 4.28 h.p.

## **PROBLEMS**

- 16. Find the horsepower of an engine whose shaft rotating at 90 r.p.m. produces a pull of 1800 lb. at the end of a brake arm 9 ft. long.
  - 17. In a brake test on a motor it was found that with a speed

of 750 r.p.m., a pull of 20 lb. was produced at the end of a 5-ft. lever. What horsepower did the motor deliver?

To find the pull produced at the end of a given lever at a given speed and horsepower we have by transposing the horsepower rule.

$$P = \frac{\text{horsepower} \times 33,000}{2\pi L \times \text{r.p.m.}}$$

Writing this equation for the length of lever arm "L" we have

$$L = \frac{\text{horsepower} \times 33,000}{P \times 2\pi \times \text{r.p.m.}}$$

and for the revolutions per min., we have

Revolutions per min. = 
$$\frac{\text{horsepower} \times 33,000}{P \times 2\pi L}$$

These rules enable us to find any one of the variable quantities in the rule in terms of the other quantities.

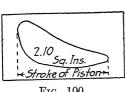
#### **PROBLEMS**

- 18. What pull "P" is produced on the end of a 4-ft. brake lever when the machine pulley is making 250 r.p.m. and machine is delivering 8 h.p.?
- 19. Find the brake horsepower of a steam engine running at 250 r.p.m. and exerting a pull of 80 lb. at the end of a lever 6 ft. long?
- 20. What length of lever arm is required for a pull of 18 lb., with a speed of 180 r.p.m. and a load of 2 h.p.?

The brake horsepower is the power of an engine or motor or water wheel available for use at the machine pulley and does not include the power lost in the machine itself through friction.

If in a steam engine we found the horsepower developed in the *cylinder* and called the *cylinder* or *indicated* horsepower we would find that this power is greater than the *brake* horsepower by the amount of power lost in friction in the engine mechanism between the cylinder and the fly-wheel or pulley. The horsepower thus lost is called the *frictional horsepower*.

In finding the indicated or cylinder horsepower of an engine we use a steam indicator or as it may be called, a recording steam gauge which takes a record of the steam pressure in the cylinder throughout the stroke of the piston. Such a card diagram or record is shown in Fig. 100. In making a test indicator cards are taken at each end of the cylinder. From these we find the average pressure throughout the stroke by finding the area of the eard, dividing this by the length of the eard, and multiplying by the constant of the indicator.



Frg. 100.

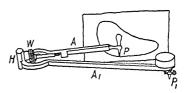


Fig. 101.

example, if the card shown has an area of 2.10 sq. in. and its length is 3.73 in., the average height  $= \frac{2.10}{3.73}$  or .563 in. If the scale of the diagram is 80 lb. to the inch this represents an average pressure of .563 imes 80 or 45 lb. per sq. in.

From this average pressure called the mean effective pressure we calculate the horsepower from the rule

$${\rm Horsepower} = \frac{PLAN}{33,000}.$$

In which P is the mean effective pressure. L is the length of stroke in ft. A is the area of the piston in sq. in. N is the number of strokes per min.

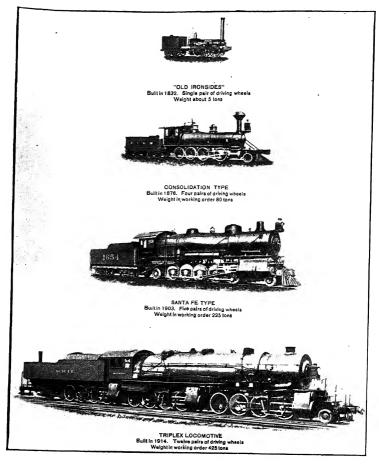


Fig. 102.—Cut showing the development in size and weight of locomotives.

To find the area of the indicator card shown, in Fig. 100 we make use of a planimeter or plane meter as shown in Fig. 101. This consists of the two arms A and  $A_1$  hinged at H and with pin points at P and  $P_1$ . The point  $P_1$  is fixed and the point P is moved around the outline of the card as shown, while the wheel W moves and carries a scale which indicates directly the number of square inches in the area traced around when the point P has completed the outline.

# CHAPTER XIV

# CALCULATION OF BELTING. ENERGY

38. Horsepower of Belting.—When two shafts are too far apart to connect them by gears for transmitting power, belts are used. The power which a belt transmits depends on the effective pull tending to turn the driven pulley and the speed with which the belt is traveling. The pull which a belt can safely stand depends on its material, width and thickness. They are sometimes made of one, two, three or four thicknesses of leather and are called single, double, triple or quadruple. The widths of belts vary from about an inch to 2 ft. depending on the amount of power transmitted. Whether single, double or other thickness of belting is used depends largely on the size of pulleys on which the belt runs. Practice shows that single belts are best on pulleys less than a foot in diameter, and double belts on pulleys a foot or larger in diameter. On main drives or in cases where the amount of power transmitted would cause an excessively wide double belt a triple or quadruple belt is used.

It has been found that single leather belts will operate satisfactorily with an allowable pull of about 30 lb. per in. of width. For a double belt about 65 or 70 lb. is allowed per in. of width. Almost any mechanical handbook contains figures taken from the results of practice for the best allowable sizes of belts.

Practice shows that the linear speed of belts should not exceed 4500 ft. per min. if the belt is to make good contact with the pulleys. As usually operated, speeds below this value are used.

If P is the allowable pull per in. of width of belt, W the width of belt in in., the pull times the width, that is  $P \times W$ , gives the total effective pull or the force transmitted by the belt. This force in lb. multiplied by the speed of the belt in ft. per min. gives the footpounds of work per min. transmitted by the belt.

To obtain horsepower we divide foot-pounds per min. by 33,000, since 1 h.p. is the performance of 33,000 ft.-lb. of work per min. If S is the speed of the belt in ft. per min. and using the letter given above for the effective pull on the belt, we have the rule

Horsepower transmitted by belt = 
$$\frac{P \times W \times S}{33,000}$$
.

Example.—Find the horsepower transmitted by a belt 5 in. in width, if the allowable pull per in. of width is 35 lb. and the speed of the belt 2400 ft. per min.

Solution:

Horsepower = 
$$\frac{P \times W \times S}{33,000}$$
 or  $\frac{35 \times 5 \times 2400}{33,000}$ 

which equals 12.7.

To find the width of belt to transmit a given horsepower for a given belt speed, we transform the above equation to express W in terms of the other quantities involved as follows.

If H represents the horsepower transmitted we have as above,  $H = \frac{P \times W \times S}{33,000}$  and therefore  $W = \frac{H \times 33,000}{P \times S}$ .

Example.—Find the width of double leather belt re-

quired to transmit 100 h.p., if the speed of belt is 4000 ft. per min. and the allowable pull per in. of width is taken as 70 lb. since this is a double belt.

Solution:

$$W = \frac{H \times 33,000}{P \times S} \text{ or } \frac{100 \times 33,000}{70 \times 4000} \text{ which equals } 11.8$$

in. Therefore a 12-in. belt should be used.

## PROBLEMS

1. Find the width of a single belt required to transmit 95 h.p. when belt runs on a 14 ft. diam. pulley at 70 r.p.m. when P = 40 lb.

NOTE.—The speed of the belt in ft. per min. equals the product of the circumference of the pulley in ft. by the r.p.m. of the pulley.

- 2. Find the width of belt required to transmit 15 h.p., if belt speed is 3500 f.p.m. and allowable pull per in. of width of belt is 50 lb.
- 39. Definitions on Energy, and Problems.—Energy is power to do work.

All objects possess energy on account of work having been done upon them at some time. Energy, like work, is measured in foot-pounds. There are, in general, two kinds of energy due either to the motion or to the position of objects. Energy of motion is called kinetic energy and energy of position is potential energy. For example, an object set in motion can overcome a certain amount of resistance before being brought to rest, and the energy which the object has on account of its motion is used up in overcoming the resistance, bringing the object to rest. Fly-wheels on engines both receive and give up energy and thus cause the engine to run more smoothly throughout the stroke.

Elevated weights have power to do work on account

of their elevated position, as in various types of hammers, pile drivers, etc. The rule for finding the kinetic energy (K.E.) of an object is:

$$K.E. = \frac{Wv^2}{64.4}$$

In which W is the weight of the object in lb., v is its velocity or speed in ft. per sec. Thus a weight of 230 lb. moving with a speed of 1200 ft. per sec. has a kinetic energy of  $\frac{230 \times 1200 \times 1200}{64.4}$  or approximately 5,140,000 ft.-lb. The rule for potential energy (P.E.) is:

$$P.E. = W \times H$$

W is the weight of the object in lb. and H is its height in ft. above a given point. The energy is then given in foot-pounds. For example: A 2-ton steam hammer when raised 4 ft. has a potential energy of  $2 \times 2000 \times 4$  or 16,000 ft.-lb. on account of its position and if let fall it would do 16,000 ft.-lbs. of work.

#### **PROBLEMS**

- 3. A weight of ½ ton is moving with a speed or velocity of 5 ft. per sec. How much energy in foot-pounds is stored up in it?
- **4.** The rim of a fly-wheel weighing 3 tons moves with a speed of 20 ft. per sec. How many foot-pounds of energy are stored up in it?
- 5. A 3-ton hammer is raised and dropped a distance of 8 ft., 8 times per min. How much work does it do per min.?
- **6.** Three thousand gallons of water are pumped into a tank an average height of 10 ft, above the level of a track. What energy does this body of water have with respect to the track?
- 7. If a punch makes a hole through a <sup>1</sup><sub>2</sub>-in, steel plate and meets with an average resistance of 30 tons, how much energy is consumed in the punching alone?
  - 8. A machinist wields a 14-lb, hammer and strikes with a veloc-

ity of 30 ft. per sec. and 28 blows per min. How many footpounds of energy are produced per min.? At what horsepower does he work?

- 9. How much work in foot-pounds is required to raise a 10-ton casting vertically through a height of 5 yd.?
- 10. If 10,000 cu. ft. of water flow per min. over a fall 20 ft. high, how many foot-pounds of energy does this water deliver at the foot of the fall? (1 cu. ft. of water weighs 62.5 lb.)

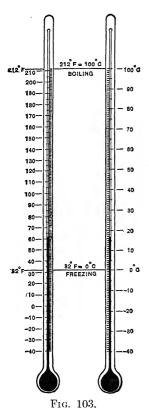
# CHAPTER XV

# HEAT

- 40. Definitions.—Heat is one form of energy. Other forms of energy such as electrical, chemical, energy of position and of motion can be changed into heat. Heat is produced by friction, by chemical action, and by the electric current flowing through a resistance such as copper or iron wire. In lighting a match heat is produced both by friction, and by chemical action in the burning of the match. The heat produced by a cutting tool on the metal being turned off in a lathe, and also that produced by friction, or in other words, by the change of mechanical work into heat. The burning of fuel is an example of a chemical change involving heat action. Objects in motion become heated when brought to rest by friction or by striking other objects.
- 41. Temperature.—The temperature of an object means not only whether the object is "hot" or "cold," but whether it takes heat from, or gives heat to, surrounding objects. When two objects at different temperatures are brought together there is a tendency toward an equalization of temperature. Like water which flows from a point of high to one of low level, heat flows from an object of high to one of low temperature. For measuring temperature, in most cases for ordinary work,

mercury thermometers are used, either the Fahrenheit or the Centigrade as shown in Fig. 103.

The Centigrade thermometer has 100 equal divisions on its scale between the point of melting ice and boiling



water, with the 0 mark at the point of melting ice.

The Fahrenheit thermometer has 180 equal divisions on its scale between the points where ice melts and water boils. The 32-degree mark is placed at the point where ice melts and the 212-degree mark at the point where water boils.

To change from a temperature in degrees Fahrenheit to the corresponding temperature in degrees Centigrade, and vice versa, the following rules are used.

F.° = C.° 
$$\times \frac{9}{5} + 32$$
.  
C.° = (F.° - 32) $\frac{5}{9}$ 

For example, using the first rule, suppose we wish to find the temperature Fahrenheit corresponding to 30° Cent., we have

F.° = 
$$30^{\circ} \times \frac{9}{5} + 32$$
  
or F.° =  $54 + 32$  or  $86^{\circ}$ .

That is, 86° on the Fahrenheit thermometer correspond to 30° on the Centigrade thermometer.

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Working the other way round and using the second rule

$$30 = (F. - 32) \frac{5}{9} \text{ or } 30 = \frac{5 F.}{9} - \frac{160}{9}$$
  
 $270 = 5 F. - 160 \text{ and } F. = 86^{\circ}.$ 

or

Alcohol and air thermometers are also used for measuring temperatures and these as well as the mercury thermometers make use of the principle of expansion and contraction of the alcohol, air or mercury with different temperatures, to obtain different readings.

42. Heat Measurement.—Heat is measured in *British Thermal Units* (abbreviated B.t.u.). One B.t.u. is the amount of heat required to raise the temperature of 1 lb. of water 1° Fahr.

For example, if 10 lb. of water are heated from  $60^{\circ}$  Fahr. to  $90^{\circ}$  Fahr. the amount of heat taken up by the water equals  $10 \times (90^{\circ} - 60^{\circ})$  or  $10 \times 30$  or 300 B.t.u.

When water reduces in temperature it of course gives up heat and the calculations are made in the same way by multiplying the weight of water in lb. by the drop in temperature in degrees Fahrenheit. This gives the number of B.t.u. liberated by the water. It should be understood that temperature means heat "intensity" or as we might say heat "pressure" and is measured in "degrees," while "quantity" of heat is measured in B.t.u. We can make the same comparison with air, in which the "intensity" or "pressure" is measured in "lb." by a gauge, while the "quantity" of air is measured in cu. ft. Again in the case of steam, we measure the "pressure" in "lb." and the "quantity" in lb. of condensed steam.

If two iron castings are of different weights, it will take

a different "quantity" of heat in B.t.u. to heat them to the same "intensity" or "temperature."

In the same way two air cylinders can contain different quantities of air and still both be at the same pressure.

If the change of temperature is given in Centigrade rather than in Fahrenheit degrees we multiply the number of Centigrade degrees by % since 1 Centigrade degree equals % Fahrenheit degrees.

If the weight is given in units other than pounds we first change the weight to the equivalent number of pounds.

Example.—How much heat in B.t.u. is taken up by 1 ton of water which is heated from 10° Cent. to 40° Cent.?

Solution.—B.t.u. = 
$$1 \times 2000 \times (40-10) \times \frac{9}{5}$$
 or 2000

$$\times \frac{30 \times 9}{5}$$
 which equals 108,000. Ans.

43. Expansion and Contraction Due to Heat.—Most substances expand or increase in volume when heated and contract or decrease in volume when heat is removed from them.

This effect takes place in solids, liquids, and gases. The principle of expansion and contraction is made of considerable use and in some cases it may cause considerable trouble unless allowed for.

When a piece of metal, for example, is heated, it expands in all directions. The expansion in one direction or dimension is that with which we are particularly concerned and is called the "linear" expansion of the substance.

For example, in the case of mercury thermometers the principle of expansion is used, also in steam piping, HEAT 109

allowance must be made for the lengthening and shortening of the pipe system due to heat changes. In laying steel rails an allowance is made between the ends of the rails for contraction and expansion. The same is true in the case of bridges and other metal structures.

The power of expansion and contraction of metal is very great and if allowance is not made for it the expansion will take place just the same with the result that the part expanding will buckle or give itself the necessary room for expansion in one way or another. These characteristics of metals are made use of in putting tires on wheels, in making shrink fits and numerous other applications.

In the case of tires put on wheels and shrink fits of various kinds, an allowance is made per in. of diameter of the object on which the fit is to be made of approximately 14000 of an in. and in the case of a locomotive driver 68 in. in diam. the tire would be turned 68 times  $\frac{1}{1000}$  or  $\frac{68}{1000}$  of an in. smaller in diam, than the center on which it is to go. It is then heated so that it will expand sufficiently to slip on to the wheel before it cools. As it cools it contracts and grips the wheel. As it cannot return entirely to its original size this gripping power is sufficient to hold the tire in place during severe service. In case of a crank to be fitted on a 6-in, shaft the crank should be turned about  $\frac{6}{1000}$  of an in. smaller than the shaft or else the shaft made 6 1000 of an in. too large and the crank made just 6 in. in diam. This, as explained will make the proper allowance so that the crank will grip the shaft tightly after it has been shrunk on.

The amount of expansion of an object depends upon the kind of material, upon its length and upon the change in temperature of the object. Tables are prepared, which give us for different materials the amount of expansion per unit length, per degree change in temperature. These figures are called "coefficients of linear expansion."

The following figures for some of the most used materials are sufficiently accurate for our purposes.

## COEFFICIENTS OF EXPANSION OF SOLIDS

Metal	Coefficient for 1° Fahr.
Aluminum (cast)	.00001234
Brass, cast	.00000957
Brass, plate	.00001052
Concrete: cement, mortar and pebbles	.00000795
Copper	.00000887
Iron, wrought	.00000648
Iron, cast	.00000556
Steel, cast	.00000636
Steel, tempered	.00000689

The coefficient for 1 Centigrade degree of temperature may be obtained from those above by multiplying them by  $\frac{9}{5}$ .

To calculate the amount of expansion the rule is, multiply the coefficient of expansion by the length of the object and by the number of degrees rise in temperature. If the length is taken in ft. the expansion will be in ft. If the length is in in. the answer will be in in., etc.

Example.—What is the amount of expansion of an 18-ft. brass rod when heated 150° Fahr.?

Solution.—Amount of expansion = Coefficient of expansion  $\times$  length of rod  $\times$  degrees rise in temperature.

Expansion =  $.0000096 \times 18 \times 150$  or .0259 ft.

Example.—What will be the increase in diam. of a 64-in. diam. locomotive driver when heated 500° Fahr.? Solution.—The coefficient of expansion of steel is taken

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as .0000064. The expansion therefore equals .0000064  $\times$  $64 \times 500 \text{ or } .2048 \text{ in.}$ 

Heat also causes liquids and gases to expand, but in the case of gases the change of volume depends upon the pressure of the gas as well as upon its temperature. Solids are liquefied by heat as in melting iron and steel and liquids are changed into vapors as in the production of steam in the locomotive boiler. 711

# PROBLEMS

1. Find the expansion in an aluminum rod 20 ft. long if it is heated 750° Fahr.

2. What is the approximate expansion in the length of a steel bridge 2800 ft. long through a difference of temperature of 0° Fahr, in winter and 90° Fahr, in summer?

- 3. The walls of a building 40 ft. wide were drawn in and straightened by using iron rods through them, placing plates on the ends of the rods, heating the rods, tightening up the plates by means of nuts and allowing the rods to cool and contract, thus drawing the building together. If the rods each time changed in temperature 800° Fahr, how much was the building drawn together with each heating and tightening of the plates?
- 4. What is the amount of expansion in a 200-ft. span of copper wire which undergoes a change of temperature of 80° Fahr.?
- 5. An iron steam pipe changes in temperature from 180° Fahr. to 8° Fahr. What allowance must be made for the change in length if the pipe is 160 ft. long at 180° Fahr.?
- 6. If steel girders on a bridge are 60 ft. long at 20° Fahr., what is their length at 94° Fahr.
- 7. What allowance must be made for a crank to be fitted to a 5-in. diam. shaft?
- 8. Find the expansion in a 40-ft, rail between the temperature of  $-20^{\circ}$  Fahr, and  $90^{\circ}$  Fahr.
- 44. Specific Heat. —The specific heat of a substance means the number of heat units (B.t.u.) required to raise the temperature of any weight of the substance

1°, as compared with the number of heat units (B.t.u.) required to raise the temperature of the same weight of water 1°.

The following are examples of specific heat: Steam .48, iron .114, lead .0314.

This means that to raise the temperature of 1 lb. of iron 1° Fahr. requires only .114 B.t.u., whereas to raise the temperature of 1 lb. of water 1° Fahr. requires 1.0 (one) B.t.u.

**45.** Latent Heat.—Latent or "hidden heat" is that required to change the *condition* of a substance from a solid to a liquid or from a liquid to a vapor without changing its temperature.

To vaporize 1 lb. of water at 212° Fahr. to steam at 212° Fahr. at ordinary pressure requires 966 B.t.u. To melt 1 lb. of ice at 32° Fahr. to water at 32° Fahr. requires 142 B.t.u. of heat.

46. Mechanical Equivalent of Heat.—The mechanical equivalent of heat means the relation between units of work in foot-pounds, and equivalent heat units. We all know that friction produces heat and to overcome the friction work must be done. When 778 ft.-lb. of work are done, an amount of heat is produced equal to 1B.t.u. and this amount of heat is sufficient to raise the temperature of 1 lb. of water 1° Fahr.

Example.—How much heat is produced by 10,000 ft.-lb. of work?

Solution:  $\frac{10,000}{778}$  or 12.8 B.t.u. Ans.

47. Heat Produced by the Electric Current.—The heat in B.t.u. produced by an electric current flowing through a resistance is equal to the square of the cur-

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rent in amperes times the resistance of the circuit in ohms times the time in sec. during which the current flows and times .00095 or

Heat in B.t.u. = (the current in amperes)<sup>2</sup>  $\times$  resistance in ohms  $\times$  seconds  $\times$  .00095

Example.—How much heat is produced by a current of 10 amperes flowing for 15 min. through a resistance of 22 ohms?

B.t.u. = 
$$10 \times 10 \times 22 \times 15 \times 60 \times .00095$$
 or 1881.  
Ans.

Working the other way round we can find the current required to produce a given amount of heat when flowing for a given time through a given resistance.

If H is the heat in B.t.u., I the current in amperes, R the resistance in ohms, and T the time in sec., the rule given above may be written,

$$H = I^2 \times R \times T \times .00095.$$

To find the current "I" in terms of the other quantities we have

$$I^2=rac{H}{R imes T imes .00095}$$
 and therefore  $I=\sqrt{rac{H}{R imes T imes .00095}}$ 

Example.—What current in amperes is required to produce 5000 B.t.u. in 10 min. when flowing through a resistance of 40 ohms?

Solution:

$$I = \sqrt{\frac{5000}{40 \times 10 \times 60 \times .00095}} \text{ or } \sqrt{\frac{5000}{24,000 \times .00095}}$$
This equals  $\sqrt{\frac{5000}{22.8}}$  or 14.8 amperes.

To find "R" from the above rule in terms of the other quantities we have

$$R = \frac{H}{I^2 \times T \times .00095}$$

$$T = \frac{H}{I^2 \times R \times .00095}$$

Similarly

#### **PROBLEMS**

- 9. A Centigrade thermometer in a boiler room reads 40° what is the corresponding temperature Fahrenheit?
- 10. The difference between the temperature of two objects is 20 Fahr. degrees. Express this difference in Cent. degrees.
- 11. What effect would there be in the readings of a mercury thermometer if the tube were not of uniform bore?
- 12. A Fahrenheit thermometer gives the temperature of water as 95°. What is the corresponding temperature in degrees Cent.?
- 13. How much heat is required to raise 100 lb. of water 10° Fahr.?
- 14. If 10 cu. ft. of water change in temperature from 90° Fahr. to 75° Fahr., how many heat units are liberated?
- 15. It is desired to raise 5000 lb. of water from 60° Fahr. to 90° Fahr. and supposing 10 per cent. of the heat value of the coal to be used, how much coal is required if there are 12,000 B.t.u. per lb.?
- 16. How much heat is required to raise 100 lb. of iron from 70° Fahr. to 350° Fahr.? To how many foot-pounds of energy does this correspond?
- 17. Which requires more energy to raise the temperature of 10 lb. of iron from 0° to 100° Fahr. or to lift 5 tons of iron 1 ft. vertically?
- 18. If 1 lb. of coal contains 12,000 B.t.u. to what height could a 1200-ton weight be raised by utilizing all the heat value of the coal?
- 19. An engine raises a casting weighing 1½ tons through a vertical distance of 5 ft. What is the number of B.t.u. which this amount of work represents?

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- 20. How many B.t.u. are required to raise 50 lb. of lead from 72° Fahr. to 520° Fahr.?
- 21. A car weighing 15 tons is lifted by a crane through a vertical distance of 10 ft. Find the number of foot-pounds of work done and the number of B.t.u. which the work represents.
- 22. A certain coal gives 14,000 B.t.u. per lb. To what height would 5 lb. of this coal lift a weight of 8 tons if 40 per cent. of the B.t.u. were utilized?
- 23. If 2 tons of water are used per hr. in an ice plant in freezing water from an average of 70° Fahr., how many B.t.u. are extracted from this water per day of 10 hr.?
- 24. Which requires the more energy, to raise the temperature of 20 lb. of lead 100° Fahr. or to lift 4 tons of iron 12 ft. vertically?
- 25. If 1 lb. of coal contains 12,000 B.t.u. to what height could a 10-ton weight be lifted, if 5 per cent. of the heat value of the coal were utilized?
- 26. A wire cage weighing 1½ tons is lifted 900 ft. four times an hr. How many foot-pounds of work are done on the cage per hr.?
- 27. If 3 per cent. of the heat value of a 12,000 B.t.u. coal were utilized, how many pounds of coal would be used per hr. in lifting the cage in Prob. 26?
- 28. Find the heat produced by a current of 20 amperes flowing for 30 min. through a resistance of 24 ohms.
- 29. How long will it take a current of 10 amperes to produce 6000 B.t.u. when flowing through a resistance of 60 ohms?
- **30.** What resistance is necessary in order to develop 4800 B.t.u. in 30 min. with a current of 8 amperes?

# CHAPTER XVI

# COMMON OR BRIGG'S LOGARITHMS

48. Definitions.—It is a good thing to become familiar with the use of logarithmic tables and common rules for using the logarithms of numbers.

In working out problems with logarithms we make use of a table of logarithms from which we can get the logarithm of any number of three, four or five figures, depending on how many figures the tables are made to give.

A logarithm of a number is expressed in two parts, one called the *characteristic* and the other the *mantissa*. The *characteristic* is obtained directly from the number and in the logarithm is the figure to the left of the decimal point. The *mantissa* is found in the tables and is the figure to the right of the decimal point in the logarithm.

49. The Characteristic.—When the number whose logarithm we are finding is greater than one the characteristic is positive, or plus. When the number is less than one the characteristic is negative, or minus. The characteristic is determined from the number of figures which make up the number whose logarithm we are finding, and is always one less than the total number of figures to the left of the decimal point. For instance, the characteristic of the logarithm for the number 3354 is 3 because there are four figures in the number. The characteristic of the logarithm for the number 384 is 2, for 24.43 it is 1, and for 7 is 0. There are all positive characteristics as the numbers were all greater than 1.

Table 1.—Common Logarithms

N.	0	1	2	3	4	. 1	5	6	7	8	9
10 11 12 13 14	0000 0414 0792 1139 1461	0043 0453 0828 1173 1492	0086 0492 0864 1206 1523	0128 0531 0899 1239 1553	0170 0569 0934 1271 1584	02 06 09 13 16	07 69 03	0253 0645 1004 1335 1644	0294 0682 1038 1367 1673	0334 0719 1072 1399 1703	0374 0756 1106 1430 1732
15 16 17 18 19	1761 2041 2304 2553 2788	1790 2068 2330 2577 2810	1818 2095 2355 2601 2833	1847 2122 2381 2625 2856	1875 2148 2405 2648 2878	19 21 24 26 29	$\frac{75}{30}$	1931 2201 2455 2695 2923	1959 2227 2480 2718 2945	1987 2253 2504 2712 2967	2014 2279 2529 2765 2989
20 21 22 23 24	3010 3222 3424 3617 3802	3032 3243 3144 3636 3820	3054 3263 3464 3655 3838	3075 3284 3483 3674 3856	3096 3304 3502 3602 3874	31 33: 35: 37 38:	21 22 11	3139 3345 3541 3729 3909	3160 3365 3560 3747 3927	3181 3385 3579 3766 3945	3201 3404 3598 3784 3962
25 26 27 28 29	3979 4150 4314 4472 4624	3997 4166 4330 4487 4639	4014 4183 4346 4502 4654	4031 4200 4362 4518 4669	4048 4216 4378 4533 4683	400 423 435 45-	32 33 18	4082 4249 4109 4564 4713	4099 4265 4425 4579 4728	$\begin{array}{c} 4116 \\ 4281 \\ 4440 \\ 4594 \\ 4742 \end{array}$	4133 4298 4456 4609 4757
30 31 32 33 34	4771 4914 5051 5185 5315	4786 4928 5065 5198 5328	4800 4942 5079 5211 5340	4814 4955 5092 5224 5353	4829 4969 5105 5237 5366	48- 498 511 527 537	19 50	4857 4997 5132 5263 5391	4871 5011 5145 5276 5403	4886 5024 5159 5289 5416	4900 5038 5172 5302 5428
35 36 37 38 39	5441 5563 5682 5798 5911	5453 5575 5694 5809 5022	5465 5587 5705 5821 5933	5178 5599 5717 5832 5944	5490 5611 5729 5843 5955	550 561 57: 587 590	23 10 55	5514 5635 5752 5866 5977	5527 5617 5763 5877 5988	5539 5658 5775 5888 5999	5551 5670 5786 5899 6010
40 41 42 43 44	6021 6128 6232 6335 6435	6031 $6138$ $6243$ $6345$ $6444$	6042 61 19 6253 6355 6454	$\begin{array}{c} 6053 \\ 6160 \\ 6263 \\ 6365 \\ 6464 \end{array}$	6064 6170 6271 6375 6474	607 618 628 638 648	80 84 85	6085 6191 6291 6395 6193	6096 6201 6304 6405 6503	6107 6212 6311 6115 6513	6117 6222 6325 6125 6522
45 46 47 48 49	6532 6628 6721 6812 6902	$\begin{array}{c} 6542 \\ 6637 \\ 6730 \\ 6821 \\ 6911 \end{array}$	6551 6616 6739 6830 6920	6561 6656 6749 6839 6928	6571 6665 6758 6848 6937	658 667 676 685 691	5 7 7	6590 6681 6776 6866 6955	6599 6693 6785 6875 6964	6609 6702 6791 6881 6972	6618 6712 6803 6893 6981
50 51 52 53 54	6990 7076 7160 7243 7321	6998 7084 7168 7251 7332	7007 7093 7177 7259 7310	7016 $7101$ $7185$ $7267$ $7318$	$7024 \\ 7110 \\ 7193 \\ 7275 \\ 7356$	703 711 720 728 736	8 2	7042 7126 7210 7292 7372	7050 7135 7218 7300 7380	7059 7143 7226 7308 7388	7067 7152 7235 7316 7396
N.	0	1	2	3	4	5		6	7	8	9

Table 1.—Common Logarithms.—Continued

	1.	ABLE	1.	OMMO.	1 1002	711111111	scc	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		
N.	0	1	2	3	4	5	6	7	8	9
55 56 57 58 59	7404 7482 7559 7634 7709	7412 7490 7566 7642 7716	7419 7497 7574 7649 7723	7427 7505 7582 7657 7731	7435 7513 7589 7664 7738	7443 7520 7597 7672 7745	7528 7604 7679	7459 7536 7612 7686 7760	7466 7543 7619 7694 7767	7551 7627
60 61 62 63 64	7782 7853 7924 7993 8062	7789 7860 7931 8000 8069	7796 7868 7938 8007 8075	7803 7875 7945 8014 8082	7810 7882 7952 8021 8089	7818 7889 7959 8028 8096	7896 7966 8035	7832 7903 7973 8041 8109	7839 7910 7980 8048 8116	7846 7917 7987 8055 8122
65 66 67 68 69	8129 8195 8261 8325 8388	8136 8202 8267 8331 8395	8142 8209 8274 8338 8401	8149 8215 8280 8344 8407	8156 8222 8287 8351 8414	8162 8228 8293 8357 8420	8235 8299 8363	8176 8241 8306 8370 8432	8182 8248 8312 8376 8439	8189 8254 8319 8382 8445
70	8451	8457	8463	8470	8476	8482		8494	8500	8506
71	8513	8519	8525	8531	8537	8543		8555	8561	8567
72	8573	8579	8585	8591	8597	8603		8615	8621	8627
73	8633	8639	8645	8651	8657	8663		8675	8681	8686
74	8692	8698	8704	8710	8716	8722		8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
N.	0	1	2	3	4	5	6	7	8	9

The negative characteristics for numbers less than one are determined in the same manner but they are expressed differently. Thus, the characteristic of the logarithm for the number .562 is -1 (minus one) and instead of writing -1 for the characteristic it is customary to change its form by adding 10, or a multiple of 10 to the characteristic, and then indicating the subtraction of the same number from this result. Thus a -1 characteristic is written 9. (mantissa) -10. For -10 + 9 = -1. The characteristic for the logarithm of the number .062 is -2 and is written 8. (mantissa) -10, since -10 + 9 = -1.

50. The Mantissa.—The mantissa consists of figures obtained from the tables and comprise portion of the logarithm to the right of the point or the decimal part of the logarithm and found by the use of the tables.

The total logarithm for a number is obtained by combining the mantissa read in the tables, with the proper characteristic.

51. Use of the Tables.—The tables give the mantissa of the number only, and are arranged to read the first two figures of the number of which we wish to find the logarithm under the column headed N. and the third figure of the number to the right of N. at the top of the table. For example, if we wish to find the logarithm of the number 215, look down the vertical column under N. for the first two figures of the number or 21 and then across the page horizontally to the vertical column headed with the third figure of the number or 5 and read the figures 3324 which is the mantissa of the logarithm for the number 215.

The characteristic for 215 is 2. Hence the logarithm of 215 is 2.3324.

The logarithm of the number 384 is 2.5843. (2 is the characteristic) (.5843 is the mantissa.)

The logarithm of the number .287 is -1.4579 (-1 is the characteristic) (.4579 is the mantissa).

The logarithm of the number 26 is 1.4150. (1 is the characteristic and .4150 is the mantissa.)

The logarithm of the number .236 is 9.3729 - 10. -1 or (9-10) is the characteristic and .3729 is the mantissa.

The logarithm of the number .0867 is 8.9380 - 10. -2 or (8 - 10) is the characteristic and .9380 is the mantissa.

The logarithm of 8.3 is 0.9191. 0 is the characteristic and 0.9191 is the mantissa.

52. To Find a Number Corresponding to a Logarithm.—We find the nearest logarithm to the one given and look across horizontally under column N. for the first two figures of the number and to the top of the column in which the logarithm occurs for the third figure. The characteristic will determine the number of figures to the left of the decimal point in the number. Thus: Find the number corresponding to the logarithm 2.3366. The nearest mantissa is .3365 occurring in the horizontal line opposite the number 21 in column headed N. and in the vertical column headed 7. Hence the number is 217. with the decimal point after the seven because the characteristic is 2 (the number of figures to the left of the decimal point in the number is one more than the characteristic).

There are four (4) general rules for the use of logarithms. These may be stated as follows:

# Rule 1.—To multiply two numbers:

Add their logarithms and find the number in the table corresponding to the sum of their logarithms. Thus: Multiply 261 by 885

 $\log 261 = 2.4166$   $\log 885 = 2.9469$ Sum of logs = 5.3635 Number = 231000.

Ans.

Pointing off one more figure in the number than the characteristic we have six figures before the decimal point in the answer.

## PROBLEMS FOR PRACTICE

- 1. Multiply 235 by 360. 6. Multiply .279 by 56.3.
- 2. Multiply 898 by 210. 7. Multiply .076 by .005.
- 3. Multiply 635 by 359. 8. Multiply 9.15 by 12.4.
- 4. Multiply 110 by 236. 9. Multiply .028 by .144.
- **5.** Multiply 2.56 by 304. **10.** Multiply 186 by .062.

# Rule 2.—To divide two numbers:

Subtract the logarithm of the subtrahend from the logarithm of the minuend and find the number in the table corresponding to the difference of these logs. Thus:

Divide 836 by 2.10

 $\log 836 = 2.9222$  $\log 2.10 = 0.3222$ 

Difference of logs = 2.6000 Number 398. Ans.

Since the characteristic is 2.6000 we point off there figures in the number, the decimal point being placed after the eight.

#### EXAMPLES FOR PRACTICE

11. Divide 600 by 42.	<b>16.</b> Divide 1.04 by .02.
12. Divide 489 by 5.	17. Divide 98.1 by 12.5.
13. Divide 628 by 124.	18. Divide .126 by 6.26.
14. Divide 989 by 2.5.	19. Divide .166 by 1.25.
15. Divide 9.67 by .002	<b>20</b> . Divide 7.89 by 6.32.

53. To Raise a Number to a Power.—Multiply the logarithm of the number by the exponent or power to which the number is to be raised, and find the number in the table corresponding to the product thus obtained.

Example.—Find the value of  $(384)^{2.5}$ .

Solution: 
$$\log 384 = 2.5843$$
.

 $2.5 \times 2.5843 = 6.4608$ . The number corresponding to the logarithm 6.4608 = 2890000. Ans. The nearest number corresponding to the mantissa .4608 is 289. The characteristic 6 gives seven figures to the left of the decimal point.

Example.—Find the value of (.0325)3.

Solution: Log 
$$.0325 = 8.5119 - 10$$
  
 $3 \times (8.5119 - 10) = 25.5357 - 30$   
or  $-5.5357$ 

From the tables the number nearest 5357 is 343. With a characteristic of -5 the answer is .0000343. *Example.*—Find the value of  $(.305)^7$ .

Solution: Log. 
$$.305 = 9.4843 - 10$$
  
 $7 \times (9.4843 - 10) = 66.3901 - 70$   
or  $-4.3901$ 

From the tables the number nearest 3901 is 246. With a characteristic of -4 the answer is .000246.

### PROBLEMS FOR PRACTICE

Find the value of

<b>21.</b> (628) <sup>3</sup>	<b>26.</b> (76.3).02
<b>22.</b> (988)· <sup>2</sup>	<b>27.</b> $(2.8)^{2.5}$
<b>23.</b> $(625)^{7.8}$	<b>28.</b> $(.002)^{2.18}$
<b>24.</b> (888) <sup>1/8</sup>	<b>29.</b> $(6.25)^{9.02}$
<b>25.</b> $(.867)^{2.62}$	<b>30.</b> $(0.0652)^{2.89}$

# 54. To Find a Root of a Number.

Divide the logarithm of the number by the index of root and find the number corresponding to the quont thus obtained.

Example.—Find the value of  $\sqrt[3]{286}$ .

Foliation: 
$$\log 286 = 2.4564$$
  $\frac{2.4564}{3} = 0.8188$ 

From the tables the number nearest .8188 is 659.

Vith a characteristic of 0, the answer is 6.59 that is point off one figure in the number thus bringing the imal point after the six.

Zxample.—Find the value of <sup>6</sup>√.000625.

*Solution:* Log 
$$.000625 = 6.7959 - 10$$

sefore dividing the logarithm by 6, we add and subst from the characteristic a number which is a multiple 10 and which will make the negative portion of the racteristic divisible by six without a remainder. If add and subtract 50 the logarithm 6.7959-10 benes 56.7959-60.

'hen 
$$\frac{56.7959 - 60}{6} = 9.46598 - 10$$
$$9.4660 - 10$$
$$-1.4660$$

rom the tables the number nearest 4660 is 292. 7ith a characteristic of -1 the answer is .292.

#### EXAMPLES FOR PRACTICE

<b>31.</b> $\sqrt{102}$	<b>36.</b> $\sqrt[6]{.260}$
<b>32.</b> $\sqrt[3]{3.85}$	<b>37.</b> $\sqrt[11]{4.26}$
<b>33.</b> $\sqrt[6]{360}$	<b>38.</b> $\sqrt[9]{1.02}$
<b>34.</b> $\sqrt[7]{206}$	<b>39.</b> $\sqrt[3]{12.6}$
<b>35.</b> % \ 0.0265	<b>40.</b> $\sqrt[5]{32.6}$

### MISCELLANEOUS PROBLEMS FOR PRACTICE

(To be solved by logarithms)

- **41.** Find the circumferences of circles whose diameters are as follows: 126 ft. 0 in. 7 ft.  $6\frac{3}{4}$  in. 6 ft. 11.2 in. 1720 ft. 0 in. 7890 ft. 0 in. (Circumference =  $3.14 \times D$  where D = diameter.)
- 42. What are the areas in sq. in. of circles with the following diameters: 62.2 in. 180 in. 9 ft. 6 in. 7 ft.  $11\frac{5}{16}$  in. 5 ft.  $10\frac{1}{2}$  in. (Area =  $.785D^2$  where D = diameter.)
- 43. What are the diameters in in. of circles whose areas are: 6450 sq. in.? 5010 sq. in.? 4050 sq. in.? 7370 sq. in?

(D = 
$$\sqrt{\frac{\text{area}}{0.785}}$$
 or D =  $\frac{1}{0.887}$   $\sqrt{\text{area}}$  where D = diameter.)

44. Find the horsepower of a locomotive from the following rule:

Horsepower = 
$$\frac{PLAN}{33,000} = \frac{130 \times 28 \times 24 \times 24 \times .785 \times 300 \times 2}{12 \times 33,000}$$

in which P=130 lb. per sq. in. pressure on the piston, cylinders 24 in.  $\times$  28 in., N=300 strokes per min.

Work out the following problems by logarithms, showing all the work:

- **45.**  $\frac{43.8 \times 2760 \times .97 \times 3.14}{7.85 \times 2.33 \times 42.6 \times .785}$
- **46.**  $\sqrt{294}$ .  $\sqrt{4.84}$ .  $\sqrt[3]{3270}$ .  $\sqrt[3]{8.24}$ .
- **47.**  $(29.7)^2$ .  $(436)^3$ .  $(2.86)^2$ .  $(6.78)^3$ .
- 48. Find the horsepower of a Pacific type locomotive with cylinders 24 in.  $\times$  26 in., drivers 80 in., running at 40 m.p.h., and ef-



ctive steam pressure of 160 lb. per sq. in. using the following gures:

$$\begin{aligned} \text{Horsepower} &= \frac{PLAN}{33,000} \\ \underline{160 \times 26 \times 24 \times 24 \times .785 \times 40 \times 5280 \times 12 \times 2 \times 2} \\ \underline{12 \times 33,000 \times 80 \times 3.14 \times 60} \end{aligned}$$

- 49. An axle weighs originally 1050 lb. It loses 5% per cent. of its sight when turned down. Find the weight of the material recoved, the number of cubic inches of material removed, and the ral weight of the axle. The material weighs .28 lb. per cu. in.
- **50.** Find the capacity in gal. and in cu. ft. of a cylindrical tank ft. high and 10 ft., in diam.
- 51. Design a cylindrical tank to hold 20,000 gal.
- **52.** Design a coal bin to hold 300 tons of coal (one ton ocpies about 37 cu. ft.).
- 53. How many cylindrically shaped iron castings 4 in. diam. Id 1½ ft. long can be made from 10 tons of metal. (Iron weighs 6 lb. per cu. in.)



## CHAPTER XVII

# THE MEASUREMENT OF RIGHT TRIANGLES

55. Definition.—In the work which follows are rules for finding the sides and angles of right triangles, that is, those having a 90 degree (90°) or right angle. These rules are used to work out problems in the shop work as shown by examples which follow.

The rules for working out triangles (that is, figures with three angles and also three sides) form a branch of mathematics called "trigonometry," "tri" meaning

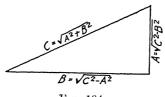


Fig. 104.

three, "gono" meaning side and "metry" measurement, or the measurement of three-sided figures. We need not think of the work necessarily as "trigonometry," but principally how it applies in working out problems that cannot be done by rules we have had before. We can use these rules also to do some of the problems quicker than by rules we have had up to this time.

56. Right Triangle Rule.—We have already had the rule that in a right triangle "the square of the hypothe-

nuse is equal to the sum of the squares of the other two sides," that is, referring to Fig. 104, if the hypothenuse is C, the height A, and the base B then

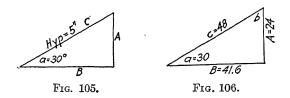
that is 
$$(\text{hyp.})^2 = (\text{height})^2 + (\text{base})^2$$

$$C^2 = A^2 + B^2 \text{ from this}$$

$$C = \sqrt{A^2 + B^2}$$
also since
$$C^2 = A^2 + B^2$$

$$A^2 = C^2 - B^2$$
and
$$A = \sqrt{C^2 - B^2}$$
and
$$A = \sqrt{C^2 - A^2}$$

That is when we have any two sides of a right triangle we can find the other side from one of these rules. It often is necessary, however, to find a side of a right triangle when we know only one other side and only one acute angle of the triangle. For example, in Fig. 105



the hypothenuse is 5 in. and the angle  $a=30^{\circ}$ . If we want to find either side A or B we cannot use the rules  $A=\sqrt{C^2-B^2}$  and  $B=\sqrt{C^2-A^2}$  since we know only one side in either of these equations. To find either side A or B we use certain ratios between the sides of the right triangle. These ratios have already been worked out for us in tables for different angles in the triangle.

If as in Fig. 106, angle  $a = 30^{\circ}$ . Side A = 24 and side C = 48.

$$\frac{\text{Side } A}{\text{Side } C} = \frac{\text{side opposite angle } a}{\text{hypothenuse}} = \frac{24}{48} = \frac{1}{2} = .5$$

57. Definition of Sine, Cosine, Tangent, and Cotangent.—This ratio of the opposite side and the hypothenuse is called the sine of angle a and for 30° it is .5. Similar ratios are worked out for all angles between 0° and 90° and put into tables for our use. If we divide the base B by the hypothenuse we have (adjacent meaning the side forming one side of the angle)

$$\frac{\text{side } B}{\text{side } C} = \frac{\text{side adjacent to angle } a}{\text{hypothenuse}} = \frac{41.6}{48} = .866.$$

This ratio is called the *cosine* of angle a. Similar ratios are worked out for all angles and put into tables for our use.

If we divide side A by side B we have

$$\frac{\text{side } A}{\text{side } B} = \frac{\text{side opposite angle } a}{\text{side adjacent to angle } a} = \frac{24}{41.6} = .577$$

This is called the tangent of angle a. Also

$$\frac{\text{side } B}{\text{side } A} = \frac{\text{side adjacent to angle } a}{\text{side opposite angle } a} = \frac{41.6}{24} = 1.73$$

This is called the *cotangent* of angle a.

These four ratios are what we will use in working out our triangles. They are called *functions* of the angle considered. Put in tabular form these functions are:

# 58. Tables of Rules for Functions of an Angle

 $\sin = \frac{\text{opposite side}}{\text{hyp.}} : \text{side opp.} = \sin \times \text{hyp.} : \text{hyp.} = \frac{\text{side opp.}}{\sin}$   $\cos = \frac{\text{adjacent side}}{\text{hyp.}} : \text{side adj.} = \cos \times \text{hyp.} : \text{hyp.} = \frac{\text{side adj.}}{\cos}$   $\tan = \frac{\text{opposite side}}{\text{adjacent side}} : \text{side opp.} = \tan \times \text{side adj.} : \text{side adj.} = \frac{\text{side opp.}}{\tan}$ 

 $\cot = \frac{\text{adjacent side}}{\text{opposite side}} \text{: side adj.} = \cot \times \text{side opp.: side opp.} = \underbrace{\frac{\text{side adj.}}{\text{cot}}}$ 

opp. = oppositehyp. = hypothenuseadj. = adjacent

59. Calculation of Functions for 30° and 45°.—Referring to Fig. 107, in which angle  $a=30^{\circ}$ , angle  $b=60^{\circ}$ , side A=10, B=17.3, and C=20.

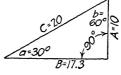
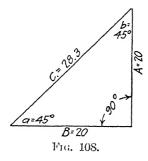


Fig. 107.

$$\sin a = \sin 30^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{10}{20} = .5$$
 $\cos a = \cos .30^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{17.3}{20} = .866$ 
 $\tan a = \tan 30^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{10}{17.3} = .577$ 
 $\cot a = \cot 30^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{17.3}{10} = 1.73$ 

the same way for Fig. 108, in which  $a = 45^{\circ}$ , b = A = 20, B = 20, and C = 28.3

$$\sin a = \sin 45^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{20}{28.3} = .707$$
 $\cos a = \cos 45^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{20}{28.3} = .707$ 
 $\tan a = \tan 45^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{20}{20} = 1.00$ 
 $\cot a = \cot 45^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{20}{20} = 1.00$ 



Explanation of Use of Tables.—These functions, ios of sides for a right triangle can be calculated for iven angle by constructing the right triangle and ming the length of its sides, but tables are prepared give us these ratios for all angles from 0° to 90° rate steps (see pages 133 to 156). These are the which we use in making calculations. In using tables the angle in degrees is read in the horizontal; the top of the page for angles up to 44° and the es in the vertical columns to the left. Angles of over are read in the horizontal line at the bottom page for degrees and the minutes in the verti-

cal columns to the right. Thus page 134 gives by minute steps, the sines and cosines of all angles, from 1° to 4° inclusive and the sines and cosines of all angles from 85° to 88° inclusive. Page 135 gives by minute steps the sines and cosines of all angles from 5° to 8° inclusive and the sines cosines of all angles from 81° to 84° inclusive. Page 145 gives the tangents and cotangents of angles from 0° to 3° and the tangents and contangents of angles from 86° to 89° and page 146 gives the tangents and cotangents of angles from 4° to 7° and tangents and cotangents of angles from 82° to 85°. Referring to page 137.

The sine of  $14^{\circ} 30' = .25038$ The sine of  $13^{\circ} 20' = .23062$ The sine of  $16^{\circ} 40' = .28680$ 

From page 143 the cosine of  $50^{\circ} 20' = .63832$  (use the right-hand column of minutes and degrees at the *bottom* of the page).

The cosine of  $73^{\circ} 50' = .27843$ . The cosine of  $0^{\circ} 15' = .99999$ . The tables of tangents and cotangents, pages 145 to 146 are read in the same way.

# **PROBLEMS**

- 1. From the tables of sines and cosines find the sines of the following angles: 15°, 30°, 60°, 90° (that is 89° 60′), 32° 10′, 43° 42′, 2° 4′, 44° 50′, 40° 55′, 45°.
  - 2. Find the cosines of the same angles given in Prob. 1.
- **3.** From the table of tangents and contangents find the *tangents* of the following angles: 14° 10′, 27° 57′, 42° 18′, 53° 12′, 63′, 28°, 45°, 30°, 17° 27′, 36°.
  - 4. Find the cotangents of the same angles given in Prob. 3.
- 5. Using the tables of functions complete the following table, finding first the angle from the function given, and then knowing the angle, find the other functions.

Prob. No.	Angle	Sine	Cosine	Tangent	Cotangent
1 2 3 4 5 6		.21644	.48226	6.31375	1.12369

# 61. Tables of Natural Sines, Cosines, Tangents, and Cotangents of Angles from $0^{\circ}$ to $90^{\circ}$ Varying by Minute Angles.

	1	00	1	11	1	00	1	11		00	
_	SINE	COSINE	1	1	SINE	COSINE	,	,	SINE	Cosine	
0	.00000	I	60	21	11000.	80000			-	-	-
I	.00020	r	50	22	.000.10	.00008	39	41	·01193	•00003	10
2	.00058	r	58	2.3		.99998	38	42	-01333	+0000 3	18
3	.00087	r	57	2.1	.00008	.99998	37	43	01251	-00003	17
4	.00116	I	50	25	.00727	-90007	36	44	·01780	-00003	16
5	.00145	r	55	20	.00750	-99997	35	45	.01300	100001	15
6	.00175	I	5.4	27	.00785	-00007	34	40	.013.38	100001	1.4
7 8	.00204	r	5.3	28	.00814	-99097	33	47	.01307	100001	1,3
8	.002,3,3	r	52	20	.00844	.00000	32	48	.01,300	-00000	13
0	.00202	1	51	30	.00873	.00000	31	40	.01425	.00000	11
10	.00201	1	50	11	1	100	.40	50	-01454	-99989	10
11	.00320	.00000	-	31	.00002	00000	20	51	-01483	.90080	o
12	.003.10	.00000	49	.3.2	.00031	.00000	28	52	.01513	.99980	કં
13	00378	.00000		3.3	00000	-90905	27	53	-01542	-90088	7
14	.00.107	.00000	47	34	.00080	400005	20	54	01571	-90088	ò
15	.00430	.00000	45	35	81010.	-90005	25	55	OUNTO.	.00087	5
16	.00.105	.00000		30	.010.17	30005	2.4	50	02010	-00087	4
17	.00.105	.00000	1.1	37 38	.01076	90004	23	57	-01658	.00086	3
18	00524	.00000	4.3		-01105	*0000t	23	58	01687	.00086	ä
10	00553	.00008	42	30	01134	10000	21	50	01716	.00085	I
30	00582	.00008	45	4,	.01104	400003	50	(1)	.01745	100085	O
	NO. NO. OF PERSONS ASSESSED.		* ' 11				11	7 1			
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,	SINE	Cosine	SINE	Cosine	SINE	COSINE	SINE	Cosine	<u>′</u>
0	.01745	.99985	.03490	-99939	.05234	.99863	.06976	-99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005	•99754	59
2	.01803	.99984	.03548	99937	.05292	.99860	.07034	-99752 -99750	58 57
3	.01832 .01862	.99983	.03577	.99936	.05321	.99857	.07092	.99738	56
4	.01802	.99982	.03635	99933	.05379	.99855	.07121	-99746	55
5	.01020	.00082	.03664	99933	.05408	.99854	.07150	99744	54
7 8	.01949	.99981	.03693	-99932	.05437	.99852	.07179	99742	53
	.01978	.99980	.03723	.99931	.05466	.99851	.07208	-99740	52
9	.02007	.99980	.03752	.99930	.05495 .05524	.99849	.07237	.99738	51 50
10	.02036	-99979	.03781	.99929		.99846	.07295	99734	49
II	.02065	.99979 .99978	.03810	.99927 .99926	.05553 .05582	.99844	.07324	.99734 .99731	48
12	.02094	99977	.03868	-99925	.05611	.00842	-07353	99729	47
14	.02152	-99977	.03897	.99924	.05640	.99841	.07382	99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.02211	.99976	-03955	.99922	.05698	.99838	.07440	-99723	44
17	.02240	-99975	.03984	.99921	.05727	.99836	.07469	.99721 .99719	43 42
	.02269 .02298	-99974	.04013	.99919	.05785	.99833	.07527	.99716	41
19 20	.02327	·99974 ·99973	.04071	-99917	.05814	.99831	.07556	99714	40
21	.02356	.99972	.04100	.99916	.05844	.00820	.07585	.99712	39
22	.02385	.00072	.04120	-99915	.05873	.99827	.07614	.99710	38
23	.02414	-99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	-99705	36
25	.02472	.99969	.04217	.00011	.05960	.99822	.07701	.99703 .99701	35
26	.02501	.99969	.04246	.99910	.05989	.99821	.07730 .07759	.99699	34 33
27 28	.02530	.99968	.04275 .04304	.99909	.06047	.99817	.07788	.00606	32
29	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	20
32	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.02705	.99963	.04449	.999oI	.06192	.99808	.07933	.99683	27 26
34 35	.02734	.99963	.04478	.99900 .99898	.06250	.00804	.07991	.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.02850	-99959	-04594	-99394	.06337	-99799	.08078	.99673	22
39	.02879	-99959	.04623	.99893	.06366	-99797	.08107	.99671 .99668	21
40	.02908	.99958	.04653	.99892	.06395	-99795	1		1
41	.02938	.99957 .99956	.04682	.99890 .99889	.06424 .c6453	·99793 ·99792	.08165	.99666 .99664	19
42 43	.02007	-99955	.04711	.00888	.06482	-99792	.08223	.9966 <b>I</b>	17
44	.03025	·99954	.04760	.99886	.06511	.99788	.08252	.99659	16
45 46	.03054	-99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.03083	-99952	.04827	.99883	.06560	.99784	.08310	-99654	14
47 48	.03112	-99952	.04856	.99882	.06598	.99782	.08339	.99652	13
40 49	.03141	.99951 .99950	.04885	.99881	.06656	.99780	.08307	.99647	II
50	.03199	•99949	.04943	.99878	.06685	.99776	.08426	-99644	10
51	.03228	.99948	.04972	.99876	.06714	-99774	.08455	.99642	9
52	.03257	-99947	.05001	.99875	.06743	-99772	.08484	-99639	8
53	.03286	-99946	.05030	-99873	.06773	.99770	.08513	.99637	6
54	.03316	-99945	.05059	.99872	.06802	-99768	.08542	.99635	5
55 56	.03345 .03374	-99944 -99943	.05088	.99870 .99869	.06831	.99766	.08571	.99632	4
	.03403	.99943	.05146	.99867	.06880	.99762	.08620	.99627	3
57 58	.03432	.99941	.05175	-99866	.06918	-99760	.08658	.99625	2
59 60	.03461	-99940	.05205	-99864	.06947	.99758	.08687	.99622	ı
-	.03490	-99939	.05234	.99863	.06976	-99756	.08716	.99619	0
1	Cosine	SINE	Cosine	SINE	COSINE	SINE	Cosine	SINE 50	1

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′	SINE	COSINE	SINE	COSINE	Sine	Cosine	SINE	COSINE	
0	.08716	.00010	.10453	.90452	.12187	-99255	.13917	.00027	бо
r	.08745	.00017	.10482	-99449	.12210	.00251	.13946	.00023	59
2	.08774	.00614	.10511	.99446	.12245	.00248	-13975	.000010	58
3	.08803	.99612	.10540	-90443	.12274	.00244	.14004	.99015	57
4	.08831	.00000	.10509	-99.140	.12302	.99240	.14033	.00011	56
5	.08860	-99607	.10507	-99437	.12331	.99237	.14061	.99000	55
5	.08880	.99604	.10626	-99434	.12300	.99233	.14090	.90002	54
7 8	.08918	.99002	.10655	.9943I	.12389	.99230	.14119	80080	53
	.08947	-99599	.10684	.99.128	.12418	.99226	.14148	-98004	52
9	.08976	.99596	.10713	.00.124	.12447	.99222	-14177	.98000	51
10	.09005	-99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.00034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.00063	.99588	.10800	.99415	.12533	.99211	.14203	.98978	48
13	.00002	.99586	.10829	.90412	.12502	.99208	.14292	-98973	47 46
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969 .98965	
15	.00150	.99580	.10887	.99406	.12620	.99200	.14340 .14378	.08061	45 44
16	.09179	.99578 -99575	.10016	.99402 -99399	.12649	.99193	.14407	.08057	43
17 18	.09237	.99572	.10043	.99390	.12706	.99180	.14436	.08053	42
10	.00206	.99570	.11002	.09393	.12735	.99186	.14464	.08048	41
20	.09295	.99507	.11031	.99390	.12704	.99182	14493	.98044	40
21	.00324	-99564	.1 1000	.99386	.12793	.99178	.14522	.08040	39
22	.00353	.00562	.11080	.90383	.12822	.90175	.14551	.08036	38
23	.00382	-99559	.11118	.90380	.12851	.99171	.14580	.98031	37
24	.00411	.99556	.11147	99377	.12880	.99107	.14008	.98927	30
25	-004.10	-99553	-11176	-9037-1	.12008	.99163	.14637	.98923	35
26	.00.100	-99551	.11205	-99370	.12037	.00100	.14666	.98919	34
27	.00,498	.99548	.11234	.00307	.12006	.90156	.14605	.98914	33
28	.00527	-995-45	.11263	.00364	.12995	.00152	.14723	.98010	32
29	.00550	.99542	.11201	.90360	.13024	.99148	.14752	.98902	31
30	.09585	.995.10	.11320	-99357	.13053	-99144	.14781		
3 r	.00614	-99537	.11340	-90354	.13081	.99141	.14810	-98897	20
32	.000.12	90534	.11378	.00351	.13110	.00137	.14838	-98893 -08889	27
33	-09071	-99531	.1 1.107	.003.17	-13130	.00133	.14867	.08884	20
34	-00700	-99528	.11,130	-90344	.13168	.00120	.14025	.08880	2.5
35 36	.00720 .00758	-90526 -90523	.11405	.003.41	.13226	.00122	.14954	.08876	24
37	.00787	.90520	.11523	90334	.13254	.00118	.14082	.08871	23
38	.00810	.90517	.11552	.90331	.13283	4.1100.	.15011	.98867	22
39	.008.45	-00514	.11580	.00327	.1,3,312	.00110	.150.10	.98863	21
40	.00874	.00511	.1 1000	-90324	.13341	.00100	.15000	.98858	20
41	.00003	.00508	.11638	.00320	.13370	.00102	.15007	08854	10
42	.00032	.00500	.11007	.00317	.13300	80000	.15120	.98840	18
43	100001	.00503	00011.	-00314	-13427	.0000.1	.15155	-08845	17
44	.00000	.00700	.11725	.00310	-13450	100001	.15184	.08841	10
45	.10010	-00407	11754	-00307	.13485	-00087	.15212	.08830	1.5
46	.100.18	1.00.100	.11783	.00303	-13514	.00083	.15241	.08832	14
4.7	.10077	101.00	.11812	-00300	-13543	.00070 .00075	.15270	.08823	13
48	.10106	.00488	0.0811	.00207	.13572	.00073	-15327	.08818	111
50	.10104	.00482	.11808	.00:00	13020	.00007	.15350	.9881.1	10
51	10101	-00470	.11027	.00286	.13658	.00063	15385	.08800	t t
52	10102	.00470	.11056	.00283	.13057	.00050	15414	.08805	8
53	.10251	-00 170	.11030	.00203	.13716	-00055	15442	.08800	7
54	.10.270	-00-170	.12014	.00276	13744	.00051	.15.171	.08700	6
55	.10308	-00107	.1.20.13	.00272	-13773	-000.17	.15500	.08701	5
50	.10337	1,00,101	.12071	.00200	.13852	.00043	.15520	.08787	-4
57	.10300	10100	.1.2100	.00265	-13831	00030	.15557	.08782	3
58	.10305	.00.158	.12120	-00 002	-13860	.00035	.15580	.08778	2
50	40424	-00455	.12158	.00258	.13880	15000	.15615	-08773	1
tio	.10453	-00.152	.12187	.00532	13017	.000.17	.15643	.08700	0
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•	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	,
0	.15643	.98769	.17365	.98481	.19081	.98163 .98157	.20791	.97815 .97800	60
I	.15672	.98764	.17393	.98476	.19109	.98152	.20848	.97803	59 58
2	15701	.98760 .98755	.17422 .17451	.98471 .98466	.19167	.08146	.20877	.97797	57
3 4	.15730 .15758	.98751	.17479	.08461	.19195	.98140	.20005	-9779I	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
5	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778 -97772	54 53
7 8	.15845	-98737	.17565	.98445	.19281	.08118	.21010	.97766	52
9	.15873	.98732 .98728	.17594 .17623	.98440 .98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	-97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49 48
12	.15988	.98714	.17708	.98420	.19423	.98096 .98090	.21132	-97742 -97735	47
13	.16017 .16046	.98709 .98704	.17737 .17766	.98414	.19452	.08084	.21180	.07729	46
14	.16074	.98700	.17794	.98404	.19509	.98079	.21218	-97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	-97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067 .98061	.21275	·97711	43
18	.16160	.98689	.17880	.98389	.19595	.98056	.21331	•97705 •97698	41
20	.16189	.98681 .98676	.17909 .17937	.98383 .98378	.19652	.98050	.21360	-97692	40
21	.16246	.98671	.17966	.98373 .98368	.19680	.98044	.21388	-97686	39 38
22	16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362 .98357	.19737	.08027	.21445	.07667	37 36
24 25	.16333 .16361	.98652	.18081	.98357	.19704	.08021	.21502	.97661	35
26	.16390	.98648	.18100	.98347	.19823	.98016	.21530	-97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642 .97636	32 31
29 30	.16476 .16505	.98633 .98629	.18195	.98331	.19908	.97992	.21644	.97630	30
31	.16533	.08624	.18252	.98320	.19965	.97987	.21672	.97623	29 28
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	
33	.16591	-98614	.18300	.98310	.20022	-97975 -97969	.21729	.97611	27
34	.16620	.98609 .98604	.18338	.98304	.20031	.97963	.21786	.97598	25
35 36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	-97592	24
37	.16706	.98595	.18424	.98288	.20136	-97952	-21843	.97585	23
38	.16734	.98500	.18452	.98283	.20165	.97946	.21871	-97579	22
39 40	.16763 .16792	.98585 .98580	.18481	.98277	.20193	.97940 -97934	.21899	.97573 .97566	21
41	.16820	.98575	.18538	.98267	.20250	.97934	.21056	.07560	20
42	.16849	.98570	.18567	.98261	.20270	.97922	.21985	-97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	-97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	-97541	16
45 46	.16935	.98556 .98551	.18652	.98245	.20364	.97905 .97899	.22070	.97534 .97528	15
47	.16002	.98546	.18710	.98234	.20421	.07893	.22126	.97521	13
47 48	.17021	-98541	.18738	.98229	.20450	.07887	.22155	-97515	12
49	.17050	.98536	.18767	.93223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51 52	.17107	.98526 .98521	.18824	.98212	.20535	.97869	.22240	.97496 .97489	8
53	.17164	.98516	.18881	.08207	.20503	.97857	.22207	-97483	
54	.17193	.98511	.18910	.98196	.20020	.97851	.22325	.97476	7
55	.17222	.98506	.18938	.98190	.20649	-97845	.22353	-97470	5
56	.17250	.98501	.18967	.98185	.20677 .20706	.97839 .97833	.22382	.97463 -97457	4
57 58	.17308	.98491	.10024	.98179	.20734	.07827	.22438	-97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	-97444	I
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	-9743 <b>7</b>	0
,	Cosine 8	SINE 0°	Cosine 7	SINE 90	Cosine 78	SINE	Cosine 7	SINE 70	•

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,	SINE	-   0	COSINE						.082	06	5593	.27	564		126	60	
	.2249	5	97437		4192	.97	030	-25	882	.96	5585	.27	592		118	59 58	
ο	.2252	3   .	97430	.2.	4220 4249	-97	015	-25	5938	.96	5578	.27	620 648		102	57	
2	.2255		.97424 .97417		4277	-07	7008		5966		5570		676		094	56	
3	.2258	8	.07411	.2	4305		1007		5994 6022		5555		704		086	55	
4	.2263	7	.97404		4333		5994    5987		6050	.90	6547		731		078	54 53	
5	.2266	5	.97398		4362		5980		6079		6540		759 7787		062	52	
7	.2260		.97391 .97384	.2	4418	.91	6973		6107 6135	9.0	6532 6524	.2	7815	.90	054	51	
	.2272		.97378	.2	4446		6966 6959	.2	6163	.9	6517	.2	7843	.90	046	50	
10	.227	8	·97371	U	24474	1	1	1	6191	9.	6509		7871		037	49	2
11	.2280	7	-97365		24503 24531		6952 6945	.2	6219		6502		7899		5029 5021	4	
12	.228		-97358		24559	و. ا	6937		6247		6494		7927 7955		6013	4	5
13	.228		-97351 -97345	11 .:	24587		6930		26275 26303		6479	.2	7983		6005	4.	
14	.220		-97338		24615		6923		26331	1 .9	647I		8011	1 .9	5997	4	
15 16	.229	48	-9733I	1.	24644 24672		6000	.:	26359	1 .9	56463		8039 8067	1.9	5989 5981		2
17 18	.229		.97325 .97318	11 .	24700	1 .0	6902		26387	1 .9	96456 96448		8005	1 .9	5972		I
	.230	33	.073II		24728	.9	6894		26415 26443		06440	.5	8123	-9	5964	1 .	0
19 20	.230	62	.97304		24756		6887	11	2647I	1.	06433	.:	28150		5956	3	9
21	.230	90	.97298		24784		96880 96873	11 .	26500	1 .	96425	-	28178 28206		5948	3	87
22	.231		.97291		.24841	1 .	06866	11 -	26528	1	96417		28234 28234		)593I	3	6
23	.231		.97284	. 11	.24869		96858	11 -	.26556 .26584		96402	11 .	28262	.9	5923	1 3	35
24 25	.23		.97271	: 11	.24897		96851 96844		.26612		96394	-	28290	1 -9	25915	1 3	34 33
26	.23	231	.97264	F	.24925		96837	11	26640	١.	.96386	1	28318 28346		95907 95898		32
27		260	.97257	. 11	.24982	١.	96829	11	.26668		.96379 .96371		28374 28374	1	იჳგით		31
		288 316	.07244		.25010		96822	11	.26696 .26724		.96363		28402	:   -	95882	1	30
29 30		345	.9723	7	.25038	- 1	.96815	- 11	.26752		.06355		28429		95874		29 28
3:	1	373	.97230	>	.25066		.96807 .96800		.26780	5	.96347	11	.28457 .28485		95865 95857	t t	23 27
3:	.23	401	.9722	3	.25122	. 1	.06703	: 11	.26808		.96340	- 11	.28513	٤١.	05849		26
33	, I	429	.9721		.25151	:	.96786		.2686	2	.96332		.2854	έl.	95841	:	25
3 <sup>4</sup>	T   -	486	.9720	3	.25179		.96778	. 11	.2689	2	.96316	i	.2856	2	.95832 .95824		24 23
3	6 .23	3514	-9719	6	.2520		.96764		.2692	0	.96308		.2859	2	.95816	5	22
3	7   -23	3542	.9718	2	.2526	3	.96756	5	.2694		.96301		.2865		.9580	7	2 I
3		357I 3599	.9717	6	.2529	I	.96749		.2697		.9628		.2868	0	.95799	)	20
4	0 .2	3627	.9716		-2532		.96742	13	.2703	- 1	.0627	7	.2870		.9579		18
4		3656	.9716		-2534 -2537	6	.9673		.2700	0	.9626	9	.2873	0	.9578: -9577		17
	- 1	3684		18	.2540	4	.9671	9	.2708	38	.9626	1	.2879		.9576		iĠ
		3712 3740		41	.2543	2	.9671		.271	14	.9624		.2882	0	9575		15
4	5 .2	3769	.971	34	.2546	8	.9670		.271		.9623	8	.288		.9574 .9574	9	14
4	46 I -3	3797	.971	27	.2551	6	.9669	0	.2720		.9623		.280		.9573	2	12
4	17 12	3825 3853	.071	13	.2554	15	.9668		.272	20	.9621	4	.280	31	.9572	4	11
	40 l •	23882	.971	o6	.2557	13	.9666	57	.272	84	.9620	6	.289		·9571		10
	50   -	23910	971		.2500		.0666	- 1	.273		.9619		.289		.9579	27	9 8
		23938	3 .979 6 .979		.256		.966		.273	40	.961	00	.200		.0560	၁ဝ	7
		23966 2399			.256	85	.966		.273		.061	74	.200	70	.9568	31	
		2402	3 .979	72	-257		.966	38 20	.273		.061	66	.200	08	.956 .956	7.3	5 4
	55 1 .	2405		205	·257	60	.966	23	.274	152	.961		.201		.956	56	3
		.2407 .2410		05I	.257	80	.066	15	.274	120	.961		.291	82	.956	47	2
		.2413	6 .97	044	.258	26	.966	200	.27	536	.961	34	.292	200	.956		I
	59 1	.2410	4 .97	037	1250	554 382	.900			564	.901		.29	237	.956	.30	-
	60	.2419	97	030	-11		-		Co	INE	Sin	NE.	Cos	INE	SIN	ΙE	1'
	7	Cosi	NE SI	NE	Cos	INE	1 Sir 50	NE	1100	1111	740	_	[]	7	3°		ı
	1		76°		11	•	J-		,,	•							

1	1'	7 <sup>0</sup> 1	1	8° 1	1	90	11 2	0°	1
'	SINE	Cosine	SINE	Cosine	SINE	Cosine	SINE	COSINE	<u>'</u>
0 1 2 3 4 5 6 7 8 9	.29237 .29265 .29293 .29321 .29348 .29376 .29404 .29432 .29460 .29487 .29515	.95630 .95622 .95613 .95605 .95596 .95588 .95579 .95571 .95562 .95554	-30902 -30929 -30957 -30985 -31012 -31040 -31068 -31095 -31123 -31151 -31178	.95106 .95097 .95088 .95079 .95070 .95061 .95052 .95043 .95033 .95024 .95015	.32557 .32584 .32612 .32639 .32667 .32694 .32722 .32749 .32777 .32804 .32832	.94552 .94542 .94533 .94523 .94514 .94504 .94495 .94485 .94476 .94466	.34202 .34229 .34257 .34284 .34311 .34339 .34366 .34393 .34421 .34448	.93969 .93959 .93949 .93939 .93929 .93909 .93899 .93889 .93879	50 58 57 56 55 54 53 52 51 50
11 12 13 14 15 16 17 18 19	-29543 -29571 -29599 -29626 -29654 -29682 -29710 -29737 -29765 -29793	.95536 .95528 .95519 .95511 .95502 .95403 .95485 .95476 .95467	.31206 .31233 .31261 .31289 .31316 .31344 .31372 .31399 .31427 .31454	.95006 .94907 .94988 .94970 .94970 .94961 .94952 .94943 .94933 .94924	.32859 .32887 .32914 .32942 .32969 .32997 .33024 .33051 .33079 .33106	.94447 .94438 .94428 .94418 .94409 .94399 .94390 .94380 .94370 .94361	34503 34530 34557 34584 34612 34630 34666 34694 34721 34748	.93859 .93849 .93839 .93829 .93819 .93809 .93789 .93779 .93769	49 48 47 46 45 44 43 42 41
21 22 23 24 25 26 27 28 29 30	.29821 .29849 .29876 .29904 .29932 .29960 .29987 .30015 .30043 .30071	.95450 .95441 .95433 .95424 .95415 .95407 .95398 .95389 .95380	.31482 .31510 .31537 .31565 .31593 .31620 .31648 .31675 .31703	.94915 .94906 .94897 .94888 .94878 .94869 .94860 .94851 .94842 .94832	.33134 .33161 .33189 .33216 .33244 .33271 .33298 .33326 .33353 .33381	.94351 .94342 .94332 .94322 .94313 .94303 .94203 .94284 .94274	.34775 .34803 .34830 .34857 .34884 .34912 .34939 .34966 .34993 .35021	.93759 .93748 .93738 .93728 .93718 .93708 .93698 .93688 .93677 .93667	39 38 37 36 35 34 33 32 31 30
31 32 33 34 35 36 37 38 39	.30098 .30126 .30154 .30182 .30209 .30237 .30265 .30292 .30320 .30348	-95363 -95354 -95345 -95337 -95328 -95319 -95310 -95301 -95293 -95284	.31758 .31786 .31813 .31841 .31868 .31896 .31923 .31951 .31979 .32006	.94823 .94814 .94805 .94795 .94786 .94777 .94768 .94758 .94749	.33408 .33436 .33403 .33518 .33545 .33573 .33600 .33627 .33655	.94254 .94245 .94235 .94225 .94215 .94206 .94196 .94186 .94176 .94167	.35048 .35075 .35102 .35130 .35157 .35184 .35211 .35239 .35266	.93657 .93647 .93637 .93626 .93616 .93606 .93596 .93585 .93575	29 28 27 26 25 24 23 22 21
41 42 43 44 45 46 47 48 49 50	.30376 .30403 .30431 .30459 .30486 .30514 .30542 .30570 .30597 .30625	.95275 .95266 .95257 .95248 .95240 .95231 .95222 .95213 .95204 .95195	.32034 .32061 .32089 .32116 .32144 .32171 .32199 .32227 .32254 .32282	.94730 .94721 .94712 .94702 .94693 .94684 .94674 .94665 .94656	.33682 .33710 .33737 .33764 .33792 .33819 .33846 .33874 .33901 .33929	.94157 .94147 .94137 .94127 .94118 .94108 .94098 .94088 .94078	.35320 .35347 .35375 .35402 .35429 .35456 .35484 .35511 .35538	.93555 .93544 .93534 .93524 .93514 .93503 .93403 .93483 .93472 .93462	19 18 17 16 15 14 13 12 11
51 52 53 54 55 55 57 59 60	.30653 .30680 .30708 .30703 .30763 .30701 .30819 .30846 .30874 .30902	.95186 .95177 .95168 .95159 .95150 .95142 .95133 .95124 .95115	.32309 .32337 .32364 .32392 .32419 .32447 .32502 .32529 .32557	.94637 .94627 .94618 .94609 .94590 .94590 .94580 .94571 .94561	.33956 .33983 .34011 .34038 .34065 .34093 .34120 .34147 .34175 .34202	.94058 .94049 .94039 .94029 .94019 .94009 .93999 .93989 .93979 .93969	.35592 .35619 .35647 .35674 .35701 .35728 .35728 .35755 .35782 .35810 .35837	.93452 .93441 .93431 .93420 .93410 .93400 .93389 .93379 .93368 .93358	98 76 5 4 3 2 x 0
•	Cosine 7	SINE 20	Cosine 7	SINE	Cosine 7	SINE	COSINE	SINE	′

	21	0	22	o اا	23	3° []	24	io i	
	Sine	COSINE	SINE	Cosine	SINE	Cosine	SINE	COSINE	
-	.358.37	.03358	.37461	.02718	.30073	.02050	.40674	.91355	60
0 1	.35864	.03348	.37488	.02707	.30100	.92039	40700	-91343	59
2	.35891	-93337	-37515	.02007	.39127	.02028	.40727 .40753	.91331	58
3	.35918	.93327 .93316	-37542 -37569	.02075	.39180	.02005	.40780	.91307	57 50
4	·35945 ·35973	.93306	-37595	.92664	.39207	.91904	.40806	.91205	55
5	.36000	.03205	.37022	.92653	-39234	.91982	.40833	.91283	54
7 8	.36027	.93285	.37649 .37676	.92642	.39260	.91971	.40860 .40886	.01272	53 52
	.36054 .36081	.932/4	.37703	.02020	.39314	.91948	.40013	.91248	51
9	.36108	.93253	.37730	.92609	.30341	.91936	.40939	.91236	50
11	.36135	.03243	-37757	.02508	.39367	.91925	.40966	.91224	49
12	.36162	.03232	-37784	.02587	-39394 -39421	.01014	.40992	.01212	48
13	.36190	.03222	.37811 .37838	.02505	-39448	.91891	.41045	.91188	47 46
14	.36217 .36244	.03201	.37865	.02554	-39474	-91870	.41072	.91176	45
15 16	.36271	.03100	.37892	.02543	.39501	.91868	.41098	.91164	44
17	.36298	.03180	.37010 -37040	.02532 .02521	-39528 -39555	.91856 .91845	.41125 .41151	.91152 .01140	43 42
18	.36325	.93159	-37973	.02510	.39581	.91833	.41178	.91128	41
19 20	.36352	.93148	-37999	.92499	.39008	.91822	41204	.91116	40
21	.36406	.03137	.38026	.02488	30635	.91810	.41231	.91104	39
22	.36434	.03127	.38053	.02.177	-3000r	-91799	41257	.91092	38
23	.36461	.03116	.38080	.02.100	.39688 .39715	.91787	.41284	.91080 .9108	37 36
24	.36488	.03100	.38134	.02444	39741	.91764	-41337	.91056	35
25 26	.36515	.93084	.38101	.02432	.30768	.91752	.41363	-91044	34
27	.36569	.03074	.38188	.02.121	-30795	.01741	.41390 -41416	.01032	33
28	.36596	.03063	.38215	.02410	.308.48	.91718	-41443	.01008	31
29 30	.36623	.03042	.38208	.92388	.39875	.91706	-41469	.90996	30
31	.36677	.0,3031	.38295	.02377	.30002	.9160.1	-41406	.90984	20
32	.36704	.0,3020	.38322	.02300	.30028	.01683	.41522	.90972	28
3.3	.36731	.03010	.38340 .38376	.02355	-39955 -39982	.01071	-41549 -41575	.90948	27
34	.36758	.02088	.38493	.02332	.,10008	.01648	.41002	90036	25
35 36	.36812	.02078	.38430	.02331	.400.35	.01036	41628	.90924	24
37	.36839	02007	.38450	.02310	.40002	.01025	-41655 -41681	.90899	23
38	.36867	.02056	.38483	.02287	.40115	.01001	41707	.90887	21
39 40	.36804	.02035	-38537	.02270	40141	.01500	-41734	.90875	20
41	.36048	.02024	.38564	.02265	.40168	01578	-41760	.90863	10
42	.36975	.02013	38501	.02254	.40105	.01500	.41787	.00851	18
43	.37002	.02002	.38617	.02243	.40248	.01555	.41840	.00820	16
44 45	.37020	.02881	38071	.02220	10275	.01531	41866	.00814	15
46	37083	.02870	.38608	.02200		.01510	.41802	.00802	1.1
47	.37110	.02850	.38725	.02150	-40328 -40355	.01508 .01400	.41045	.00778	12
48	.37137 .3716.4	.02840 .02838	.38752	.02175	40381	61484	-41072	.00706	11
49 50	37101	.02827	38805	.02104	.,10,408	-00.472	.41908	.90753	10
51	.37218	.02810	.38832	.02152	-40434	.01461	.4202.1	.007.11	8
52	.37245	.02805	38850	.021.41	"lotor	.01440	.42077	.00720	7
53	.37272	.02704	.38886 .38912	.02130	40488 40514	.01437	.42101	.00704	6
54 55	.37200 .37320	.02773	.38030	.02107	.40541	61414	.42130	.00002	5
56	-37353	.02762	38000	.02000	10507	.01.(02	.42150	80000	3
57	.37380	.02791	.38003	.02085	15001	.01300	.42200	.00655	2
58 59	·37407 ·37434	.02740	.30020 .30040	.0.5073	.100.17	.01366	-42235	.00043	T
(io	.37461	.02718	-30073	.02050	40074	.01355	.42202	.00031	1. 2
	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	1
	GOSINE	80 3181	6	7°	1 0	66°	11 6	50	1

,	9	5° 1	1 2	6°	1 2	7°	1 2	80	l
,	SINE	Cosine	SINE	COSINE	SINE	COSINE	SINE	Cosine	
0 1 2	-42262 -42288 -42315	.90631 .90618 .90606	-43 <sup>8</sup> 37 -43 <sup>8</sup> 63 -43 <sup>8</sup> 89	.89879 .89867 .89854	·45399 ·45425 ·45451	.89101 .89087 .89074 .89061	.46947 .46973 .46999	.88295 .88281 .88267 .88254	59 58
3 4 5 6	.42341 .42367 .42394 .42420	.90594 .90582 .90569 .90557	-43916 -43942 -43968 -43994	.89841 .89828 .89816 .89803	•45477 •45503 •45529 •45554 •45580	.89048 .89035 .89021 .89008	.47050 .47076 .47101 .47127	.88240 .88226 .88213	57 59 55 54 53
	.42446 .42473 .42499 .42525	•90545 •90532 •90520 •90507	.44020 .44046 .44072 .44098	.89790 .89777 .89764 .89752	.45606 .45632 .45658	.88995 .88981 .88968	•47153 •47178 •47204	.88185 .88172 .88158	52 51 50
11 22 13 14 15 16 17 18	.42552 .42578 .42604 .42631 .42657 .42683 .42709 .42736 .42762	-90495 -90483 -90470 -90458 -90446 -90433 -90421 -90408 -90396	.44124 .44151 .44177 .44203 .44229 .44255 .44281 .44307 .44333	.89739 .89726 .89713 .89700 .89687 .89674 .89662 .89649 .89636	-45684 -45710 -45736 -45762 -45787 -45813 -45839 -45865 -45891	.88955 .88942 .88928 .88915 .88902 .88888 .88875 .88862 .88848	.47229 .47255 .47281 .47306 .47332 .47358 .47383 .47409 .47434	.88144 .88130 .88117 .88103 .88089 .88075 .88062 .88048	49 48 47 46 45 44 43 42 41
20 21 22 23 24 25 26 27 28	.42788 .42815 .42841 .42867 .42894 .42920 .42946 .42972 .42999	.90383 .90371 .90358 .90346 .90334 .90321 .90309 .90296 .90284	•44359 •44385 •44411 •44437 •44404 •44490 •44516 •44542 •44568	.89623 .89610 .89597 .89584 .89571 .89558 .89545 .89532 .89519	.45917 .45942 .45968 .45994 .46020 .46046 .46072 .46097 .46123	.88835 .88822 .88808 .88795 .88782 .88768 .88755 .88741 .88728	.47460 .47486 .47511 .47537 .47562 .47588 .47614 .47639 .47665	.88020 .88006 .87993 .87979 .87965 .87951 .87937 .87923 .87909	40 39 38 37 36 35 34 33 32
29 30 31 32	.43025 .43051 .43077 .43104	.90271 .90259 .90246 .90233	.44594 .44620 .44646 .44672	.89506 .89493 .89480 .89467	.46149 .46175 .46201 .46226	.88715 .88701 .88688 .88674	.47690 .47716 .47741 .47767	.87896 .87882 .87868 .87854	31 30 20 28
33 34 35 36 37 38 39 40	.43130 .43156 .43182 .43209 .43235 .43261 .43287 .43313	.90221 .90208 .90196 .90183 .90171 .90158 .90146 .90133	.44698 .44724 .44750 .44776 .44802 .44828 .44854 .44880	.89454 .89441 .89428 .89415 .89402 .89389 .89376 .89363	.46252 .46278 .46304 .46330 .46355 .46381 .46407 .46433	.88661 .88647 .88634 .88620 .88607 .88593 .88580 .88566	.47793 .47818 .47844 .47869 .47895 .47920 .47946 .47971	.87840 .87826 .87812 .87798 .87784 .87770 .87756 .87743	27. 26 25 24 23 22 21 20
41 42 43 44 45 46 47 48 59	-43340 -43366 -43392 -43418 -43445 -43471 -43497 -43523 -43549 -43575	.90120 .90108 .90095 .90082 .90070 .90057 .90045 .90032 .90019 .90007	.44906 .44932 .44958 .44984 .45010 .45036 .45062 .45088 .45114 .45140	.89350 .89337 .89324 .89311 .89298 .89285 .89272 .89259 .89232	.46458 .46484 .46510 .46536 .46561 .46587 .46613 .46639 .46664	.88553 .88539 .88526 .88512 .88499 .88485 .88472 .88458 .88445 .88431	.47997 .48022 .48048 .48073 .48099 .48124 .48150 .48175 .48201	.87729 .87715 .87701 .87687 .87673 .87659 .87645 .87631 .87617	19 18 17 16 15 14 13 12 11
51 52 53 54 55 56 57 58 59	.43602 .43628 .43654 .43680 .43706 .43733 .43759 .43785 .43811 .43837	.89904 .89981 .89968 .89956 .89943 .89930 .89918 .89905 .89870	.45166 45192 -45218 -45243 -45269 -45295 -45321 -45347 -45373 -45399	.89219 .89206 .89193 .89180 .89167 .89153 .89140 .89127 .89114	.46716 .46742 .46767 .46793 .46819 .46844 .46870 .46896 .46921 .46047	.88417 .88404 .88390 .88377 .88363 .88349 .88336 .88322 .88338 .88295	.48252 .48277 .48303 .48328 .48354 .48379 .48405 .48430 .48456 .48481	.87589 .87575 .87561 .87546 .87532 .87518 .87504 .87490 .87476 .87462	98 76 5 4 3 2 1
,	COSINE	Sine 4°	Cosine 6	SINE	Cosine 62	STATE	Cosine 6	SINE	<del>,</del>

2	0.0	"	30	10 11	. 31	0 1	32	0 1	
/	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	<u>'</u>
0	-48481	.87462	.50000	.86603	.51504	.85717	-52992	.84805 .84789	60 59
1	-48506	.87448	.50025	.86588	-51529	.85702	-53017	.84774	58
2	.48532	.87434	.50050	.86573	-51554	.85687	-53041 -53066	.84759	57
3	.48557 .48583	.87420	.50076	.86559	-51579	.85672 .85657	.53001	.84743	56
4	-48583	.87406	.50101	.86544	.51604 .51628	.85642	.53115	.84728	55
5	48608	.87391	.50126	.86530	.51653	.85627	.53140	.84712	54
2	-48634	87377	.50151	.86515 .86501	.51678	.85612	.53164	.84607	53
7 8	.48659 .48684	.87363	.50176	.86486	.51703	.85597	.53189	.84681	52
9	48710	.87349	.50201	.86471	.51728	.85582	-53214	.84666	51
10	48735	87335	.50252	.86457	.51753	.85567	.53238	.84650	50
11		.87321	-		.51778	85551	.53263	.84635	49
12	·48761	.87306	-50277	.86442 .86427	.51773	.85536	.53288	.84619	48
13	.48786 .48811	.87292	.50302	.86413	.51828	.85521	.53312	.84604	47
14	40011	.87278	.50327	.86398	.51852	.85506	•53337	.84588	46
7.5	.48837 .48862	.87264 .87250	.50352 .50377	.86384	.51877	.85491	.53361	84573	45
15	.48888	.87235	.50403	.86369	.51902	-85476	.53386	-84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
17	.48938	.87207	.50453	.86340	.51952	.85446	-53435	.84526	42
19	.48964	.87193	.50478	.86325	-51977	.8543I	.53460	84511	41
20	48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	·40014	87164	.50528	.86295	.52026	.85401	-53509	.84480	39
22	-49040	87150	.50553	.86281	.52051	.85385	-53534	.84464	38
23	49065	.87136	.50578	.86266	.52076	.85370	-53558	.84448	37
24	49090	.87121	.50603	.86251	.52101	-85355	-53583	.84433	36
25	49116	.87107	.50628	.86237	.52126	85340	-53607	.84417	35
26	49141	.87093	.50654	.86222	.52151	.85325	-53632	-84402	34
27	49166	.87079	.50679	.86207	-52175	.85310	.53656	.84386	33
28	49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	49217	.87050	.50729	.86178	-52225	.85279	-53705	-84355	31
30	49242	87036	-50754	.86163	•52250	85264	·5373°	.84339	30
3r	·49268	.87021	.50779	.86148	-52275	85249	-53754	.84324	20
32	49293	87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	49318	.86993	.50829	.86119	-52324	.85218	-53804	.84292	27
34	-49344	.86978	.50854	.86104	-52349	.85203	.53828	.84277	26
35	-49369	86964	.50879	.86089	.52374	.85188	-53853	.84261 .84245	25 24
36	•49394	.86949	.50904	.86074	.52399	.85173	-53877	.84230	23
37	•49419	.86935	.50929	.86059	.52423	.85157 .85142	.53902 .53926	.84214	22
38	-49445	.8692I	-50954	.86045	.52448	.85142	.53920 .53951	.84108	21
39	49470	.86906	-50979	.86030 .86015	.52473	.85112	-53975	.84182	20
40	-49495	.86892	.51004		1			.84167	1
41	49521	.86878	.51029	.86000	-52522	.85096	.54000	.84151	18
42	.49546	.86863	.51054	.85985	.52547	.85081 .85066	.54024 .54049	.84135	17
43	·49571	-86849	.51079	.85970	.52572	.85051	.54073	.84120	16
44	.40596	.86834	.51104	.85956 .85941	.52621	.85035	.54097	.84104	15
45	.49622	.86820 .86805	.51129	.85926	.52646	.85020	.54122	.84088	14
46	-49647	.86791	-51179	.85011	.52671	.85005	.54146	.84072	13
47 48	.49672 .49697	.86777	-51204	.85896	.52696	.84989	.54171	.84057	12
	-49723	.86762	-51229	.85881	.52720	.84974	.54195	.84041	11
40	.49748	.86748	-51254	.85866	-52745	.84959	.54220	.84025	10
			.51270	.85851	.52770	.84943	.54244	.84009	9
5 r	-49773	.86733 .86719	.51279	.85836	.52794	.84928	.54269	.83994	8
52	.40798	.86704	.51329	.85821	.52819	.84013	.54293	.83978	7
53 54	.40849	.86690	-51354	.85806	.52844	.84897	-54317	83962	6
55	.40874	.86675	-51379	.85792	.52869	.84882	-54342	.83946	5
50	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83030	4
57	49924	.86646	.51420	.85762	.52018	84851	-54301	.83015	3
38	.49950	.86632	-51454	.85747	-52943	.84836	.54415	.83800	
50	.49975	.86617	-51479	85732	.52967	.84820	.54440	.83883	1
tio	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
-		1-5	Const	Crass	COSINE	SINE	COSINE	SINE	1
1	COSINE	OO SINE	COSINE	9° SINE	COSINE	8°	COSTAL	70	1
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•	SINE	Cosine	SINE	Cosine	SINE	COSINE	SINE	Cosnie	
0	-54464	.83867	-55919	.82904	-57358	.81915 .81899	.58779 .58802	.80902 .80885	60
2	.54488 •54513	.83851 .83835	·55943 ·55968	.82887 .82871	.57381 .57405	.81882	.58826	.80867	59 58
3	-54513	.83819	.55992	.82855	.57429	.81865	.58849	85850	57
4	.5456I	.83804	.56016	.82839	•57453	.81848 .81832	.58873 .58896	.80833 .8081ú	56
5	.54586 .54610	.83788 .83772	.56040 .56064	.82822 .82806	-57477 -57501	.81815	.58020	.80799	55 54
7 8	.54635	83756	-56088	.82790	-57524	.81798	.58043	80782	53
	-54659	.83740	.56112	.82773	-57548	.81782 .81765	.58967	.80765 .80748	52 51
9	.54683 .54708	.83724 .83708	.56136 .56160	.82757 .82741	·57572 ·57596	.81748	.59014	.80730	50
ıı	-54732	83692	.56184	.82724	.57619	.81731	-59037	.80713	49
12	.54756	.83676	.56208	.82708 .82692	.57643	.81714	.59061 .59084	.80696 .80679	48 47
13	.54781	.83660 .83645	.56232 .56256	.82092	.57667 .57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	-57715	.81664	.59131	-80644	45
16	.54854	.83613	.56305	.82643 .82626	-57738	.81647 .81631	-59154 -59178	.80627 .80610	44 43
17	.54878 .54002	.83597 .83581	.56329 .56353	.82020	-57762 -57786	.81614	.50201	.80593	43
19	•54927	-83565	-56377	.82593	-57810	.81597	.59225	.80576	41
20	·54951	-83549	.56401	.82577	-57833	.81580	-59248	.80558	40
2 I 2 2	-54975	-83533	-56425	.82561 .82544	.57857 .57881	.81563 .81546	-59272	.80541 .80524	39 38
23	-54999 -55024	.83517 .83501	.56449 .56473	.82528	.57904	.81530	.59318	.80507	37
24	-55048	-83485	.56497	.82511	.57928	.81513	-59342	.80489	36
25 26	-55072 -55097	.83469	.56521 .56545	.82495 .82478	·57952 ·57976	.81496 .81479	.59365	.80472 .80455	35 34
27	.55121	.83453 .83437	.56569	.82462	.57999	.81462	-59412	.80438	33
27 28	-55145	.83421	.56593	82446	.58023	.81445	.59436	80420	32
29 30	-55169	.83405 .83389	.56617 .56641	.82429	.58047	.81428 .81412	.59459 .59482	.80403 .80386	31
31	-55218	.83373	.56665	.82396	.58004	.81395	.59506	.80368	20
32	-55242	83356	.56680	.82340	.58118	.81378	-59529	.80351	28
33	-55266	83340	-56713	.82363	.58141	.81361	-59552 -59576	.80334 .80316	27 26
34	.55291 .55315	.83324 .83308	.56736 .56760	.82347 .82330	.58165	.81344 .81327	-59599	.80200	25
35 36	-55339	83202	.56784	.82314	.58212	.81310	.59622	.80282	24
37 38	-55363	83276	.56808	.82297 .82281	.58236	.81293	.59646 .59669	.80264 .80247	23
39	.55388 .55412	.83260 .83244	.56832 .56856	.82264	.58283	.81250	-59693	.80230	21
40	-55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	-55460	83212	-56904	.82231	-58330	.81225	-59739	.80195	18
42 43	-55484 -55500	.83195 .83179	.56928 .56952	.82214	.58354	.81101	-59763 -59786	.80160	17
44	-55533	.83163	.56976	.82198 .82181	.58401	.81174	.59809	.80143	
45 46	-55557 -55581	.83147	-57000	82165	.58425	.81157	.59832 .59856	.80125	15
47	-55505	.83131	-57024 -57047	.82148 .82132	.58449 .58472	.81123	.59879	.80091	13
48	-55630	-83098	-57071	.82115	.58496	.81106	-59902	.80073	12
49 50	-55654 -55678	.83082 .83066	.57095 .57119	.82098 .82082	-58519	.81089	.59926 -59949	.80056 .80038	11
51	-55702	.83050	-57143	.82065	.58567	.81055	.59972	.80021	1
52	-55726	.83034	-57167	-82048	.58590	.81038	-59995	.80003	8
53 54	-55750	.83017	-57191	.82032	.58614	.81021	.60019	.79986	7 6
	-55775 -55 <u>7</u> 99	.83001 .82085	-57215 -57238	.82015 .81999	.58637 .58661	.80087	.60065	.79951	
55 56	-55823	.82969	-57262	81982	.58684	.80970	.60089	-79934	5 4 3 2
57 58	.55847 .55871	.82953 .82936	.57286	.81965	.58708	.80953 .80936	.60112	-79916 -79899	3
59 50	.55895	.82920	•57310 •57334	.81949	.5873I .58755	.80930	.60158	.79881	I
50	-55919	-82904	-57358	.81915	.58779	.80902	.60182	.79864	0
•	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	1
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-	.60182	.79864	.61566	.788or	.62932	.77715	.64279	.76604	60
I	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586 .76567	59 58
2	.60228	-79829	.61612	.78765 .78747	.63000	.77660	.64346	.76548	57
3	.60251 .60274	.79811 -79793	.61658	.78729	.63022	.7764I	.64368	.76530	56
4	.60298	.79776	.61681	.787II	.63045	.77623	.64390	.76511	55
5	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7 8	.60344	-79741	.61726	.78676	.63000	-77586	.64435	.76473	53 52
	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455 .76436	5 I
9	.60390	.79706	.61795	.78022	.63158	.77531	.64501	.76417	50
10		.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
11	.60437 .60460	.79653	.61841	-78586	.03203	.77494	.64546	.76380	48
12 13	.60483	.79035	.61864	.78568	.63225	77476	.64568	.7636r	47
14	60506	.79618	.61887	.78550	.63248	.77458	.64590	-76342	46
15	.60529	.79600	.61909	.78532	.03271	·77439 ·77421	.64612	.76323 .76304	45 44
16	.60553	.79583	.61932	.78514 .78496	.63293	.77402	.04055	.76286	43
17	.60576 .60599	.79505 .79547	.61933	.78.178	.63338	.77384	.64679	-76267	42
10	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.02024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	-79494	.62046	.78424	.63406	-77329	.64746	.76210	39
22	.60691	-79477	.62060	.78405	.63428	.77310	.64768	.76192	38
23	.60714	•79459	.62092	.78387 .78369	.63451	.77292 .77273	.64812	.76173 .76154	37 36
24	.60738	.79441	.62115	.78351	.63496	.77255	.64834	.76135	35
25 26	.60761	.70406	.62100	.78333	.63518	.77236	.64856	.76116	34
	.60807	.79388	.62183	.78315	.63540	.77218	64878	.76007	33
27	.60830	.7937I	.62200	.78297	-63563	-77100	.6.1001	.76078	32
29	.60853	.70353	.62220	.78279 .78261	.63585	.77181	.64923	.76059 .76041	31
30	.60876	-79335	.62251		-	1	.64967	.76022	20
31	.60800	.79318	.6227.4	.78243 .78225	.63630	-77144 -77125	.64989	.70002	28
32	.60922	.70300 .70282	.02320	.78206	.03075	.77107	.65011	-75984	27
33 34	.60008	.70204	.023.42	.78188	.63698	.77088	.65033	-75905	26
35	.60001	.702.17	.62305	.78170	.63720	.77070	.05055	-75946	25
36	.61015	.70229	.62388	.78152	.637.42	.77051	.65077	-75027 -75008	24
37	.61038	.70211	.02411	.78134 .78116	.63765	.77033	.65100	.75889	22
38	.61061	-70103 -70176	.62450	.78008	.63810	.70000	.65144	.75870	21
39 40	.61107	.79158	.62479	.78070	.63832	-70977	.65166	-75851	20
41	.61130	.701.10	.62502	.78061	.63854	.76050	.65188	-75832	10
42	.61153	.79122	.02524	.780.13	.03877	-700.10	.65210	-75813	18
43	.61176	.70105	.025.17	-78025	.63800	.70g21	.65232	-75704	17
44	.61100	.70087	.02570	.78007 .77088	.63022 .63044	.76003 .76884	.65254	-75775 -75756	15
45	.61222	.70060	.02502	-77070	.03000	.76866	.65208	-75738	1.4
46 47	.61268	.70033	.62038	77052	.63080	-70847	.65320	-75719	13
48	.61201	70016	.02000	-77034	1101.0.	-70828	.65342	-75700	12
49	.61314	.78008	.62683	.77010	.0.1033	.76810	.05364	-75080	11
50	.61337	.78980	.02700	-77807	.64056	.70791	.65386	.750bi	
5 r	.61360	.78062	.62728	77870	.6.1078	.70772	.65,408	.75642 .75023	8
52	.61383	.78044 .78026	.02751	.77801 .77843	.0.1100	.70754 .70735	.05.152	.75004	
53 54	.61420	.780.20	.62774	.77824	.641.15	.76717	.05.17.4	-75585	6
55	.61451	.78801	.62810	.77800	.6.1107	.76608	.65.100	-75506	5
56	.61474	.78873	62842	-77788	.0.11.0	-70070	.05518	-75547	4
57	.61407	-78855	62864	-77700	.6.1212	-76661	.05540	.75528 .75500	3 2
58	.61520	.78837	.0.2887	-77751 -77733	.64234	.76623	.05584	.75400	1
59 60	.61543	.78801	.0.2032	-77715	.04270	.7660.1	.05000	-75-171	0
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	65759 6578 65803	8	52 .6693 .6695 .4 .669 .5 .6699 .5 .6702 .6 .6704 .7 .6706 .8 .6708 9 .6710	5 -742 6 -742 8 -742 9 -742 1 -742 3 -741 6 -711 7 -741	95 76 56 27 17 98 78	.6826 .6822 .6826 .6826 .6836 .6832 .6834 .6837	21 .73 42 .73 44 .73 85 .73 86 .73 9 .72 9 .72 1 .72	1135 096 076 056 036 016 096 076	.694 .695 .695 .695 .695 .695 .696 .696	87 08 29 49 70 91 12	-719 -718 -718 -718 -718 -718 -717 -717	14 94 73 53 33 13 92	50 50 57 50 55 54 53 52 51
10 11 12 13 14 15 16 17 18 19	.65847 .65860 .65891 .65913 .65935	7 .7526 .7524 .7522 .7520 .7518 .7516 .7516 .75140	1 .67151 1 .67172 2 .67194 3 .67215 4 .67237 6 .67258 6 .67301 7 .67323	7410 2 .7408 4 .7406 7404 7402 7400 7398 7396 7394	00 II	.6841 .6843 .6845 .6847 .68518 .6853 .6856 .6858 .68603	4 .726 5 .725 6 .725 7 .728 8 .728 0 .728 1 .727 2 .727 3 .727	377 377 37 37 97	.6969 .6969 .6973 .6973 .6975 .6986 .6984 .6986	06 17 18 19 19 10 1 2	-7173 -7171 -7169 -7167 -7163 -7161 -7159 -7156 -7154	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	50 40 48 47 40 45 44 43 42 41 40
21 22 23 24 25 26 27 28 29 30	.66066 .66088 .66109 .66131 .66153 .66175 .66197 .66240 .66240	.75050 .75030 .75011 .74992 .74973 .74953 .74934 .74915 .74896	.67366 .67387 .67409 .67430 .67452 .67473 .67495 .67516 .67538	.7390. .7388; .7386; .73840; .73826; .7386; .73767; .73747; .73728	4	.68645 .68666 .68688 .68799 .68730 .68751 .68772 .68793 .68814	.727 .726 .726 .726 .726 .726 .725 .725 .725 .725	17 97 77 57 37 17 97	.6990 .6992 .6994 .6996 .6998 .7002 .7002 .7002 .7007	4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	.71508 .71488 .71468 .71447 .71427 .71407 .71386 .71366 .71345	333	30 38 37 36 35 34 33 32 31
31 32 33 34 35 36 37 38 39 40 41	.66284 .66306 .66327 .66349 .66371 .66393 .66414 .66436 .66458	.74876 .74857 .74838 .74818 .74799 .74780 .74760 .74741 .74722 .74703	.67580 .67602 .67623 .67643 .67666 .67688 .67709 .67730 .67752	.73708 -73688 -73669 -73649 -73629 -73610 -73590 -73570 -73551 -73531		.68857 .68878 .68899 .68920 68941 68962 68983 69004 69025	.7251 .7249 .7247 .7243 .7241 .7239 .7237 .7235 .7233	777777777777777777777777777777777777777	.70112 .70132 .70153 .70174 .70195 .70215 .70236 .70257 .70277		71305 71284 71264 71243 71223 71203 71182 71162 71141 71121	2 2 2 2	20 28 27 26 25 24 23 22 21 21
42 43 44 45 46 47 48 49 50	.66501 .66523 .66545 .66566 .66588 .66610 .66632 .66653 .66675	.74683 .74664 .74644 .74625 .74606 .74586 .74567 .74548 .74528 .74509	.67795 .67816 .67837 .67859 .67880 .67901 .67923 .67944 .67965 .67987	.73511 .73491 .73472 .73452 .73432 .73413 .73393 .73373 .73353 .73353	.6 .6 .6 .6 .6	59067 59088 59109 59130 59151 59172 59193 59214 9235 9256	.7231 .7229 .7227 .7225 .72236 .72216 .72176 .72176 .72136		70319 70339 70360 70381 70401 70422 70443 70463 70484 70505	.7 .7 .7 .7 .7 .7	71100 71080 71080 71039 71019 70998 70978 70978 70978	I	3 2 1
51 52 53 54 55 56 57 58 59	.66718 .66740 .66762 .66783 .66805 .66827 .66848 .66870 .66891	.74489 .74470 .74451 .74431 .74412 .74392 .74373 .74353 .74334 .74314	.68008 .68029 .68051 .68072 .68093 .68115 .68136 .68157 .68179	.73314 .73294 .73274 .73254 .73234 .73215 .73195 .73175 .73155 .73135	.6. .6. .6. .6. .6. .6. .6. .6.	9277 9298 9319 9349 9361 9382 9493 9424 9445	.72116 .72095 .72075 .72055 .72035 .72015 .71995 .71974 .71954	-2   -3   -3   -3   -7   -7   -7	70525 70546 70567 70587 70608 70628 70649 70670 70690	.70 .70 .70 .70 .70 .70 .70	0896 0875 0855 0834 0813 0793 0772 0752		77.75
'	Cosine 48	SINE	Cosine 47	SINE		SINE 46	SINE		SINE 45	Sı	NE	,	

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0	.00000	Infinite.	.01746	57.2000	40,440.2	Milister	1617 F43	219 MES	19
1	.00020	3437-750	.01775	See, Butt	14,95.21	JN 19934	कार जेड्डाव कार्य जेव्हेंब्र	19 19 19 1	+ (6)
2	.00058	1718.870	.0180.1	55-4415	40,45,40	38 3664 27 0474	5/13/20	19 15219	1, 4
3	.00087	1145.020 850.430	.01833	54-5013	43,45,79	37.7117	A12.42.2	19 12 30	+1-
4	.00116	687.549	orSor	52.8821	0.4048	27.47000	· 在18 直衛型	京市 1.7 東市	1.1
5 6	.00175	572.057	.01020	52.0507	seighter.	27.27157	1.19.4374	27 gt 41	1.5
78	.00204	491.100	.010.10	\$1,3032	angleight.	27 115644	215445	gA green	- 12
	.00233	420.718	.01078	30.5157	314725 114754	20:345.0	2755 6	\$15 1 18	
9	.00202	381.071 343-774	.02030	49.1039	1157753	attra iste	215544	19-1-17	
10		312.521	.020fd	48.4121	0.4812	26. 2 2061	128,87-3	\$2.12% 2	4
11	.00320	280.478	.02005	47-7.409	0,034	20.014.17	321.128	17 180 1	9 "
13	.00378	204-441	.02124	47/0854	33871	24.74.44	Aug fight	\$ 7 19/18	à
14	.00407	245-552	.02153	40.44%0	43 (49.80)	45,641%	J. 1. 1. 1413	17	41
15	.004,36	220.182	.02152	45.8204	.03020	25 4547	and the	19 . 2 1	4 4
16	.00405	214.858	.02211	45.2261	40 (0/NZ	25 11705	120 7 13	1 17 4	4.5
17 18	.00495	100.08.4	.02.200	Addiddes	cause	24 Next N	309 840	27 1214	4.9
10	.00553	180.032	.0220N	41.50%1	04046	24 7185	A2+ 1234	27.31.18	9.1
20	.00582	171.885	.02328	42.9641	16.04. 14.1	24/54175	201944	23 26.25.3	-
21	.00611	163.700	.02357	42.43.45	314 1114	24 MY	pikh 4	12 -437	27
22	.00040	150.250	.02,680	41.0158	234111	24 1057	211949	27- UNF#1	2.0
23	.00000	140.465	.02415	49.0174	204164 204191	24 (12/14)		16 8114	27
24 25	.00698	143-237	.02473	49.4458	214 2 217	2114345	-1 11951	10 14000	5.
26	.00756	132.219	02502	30.0655	4142519	21 1 121	arriess.	gr. Caratta	1.4
27	.00785	127.321	.02531	30.5080	a14470	24.47426	180 184	28-18 A	2.5
28	.00814	122.774	102500	Auto 1500%	214 F.M	21 22 17	3.8× 3.19	37 Var 3	5.9
29	.00844	118.530	1037/20	48.6177 1 48.1885	014117	24/05/2	Weg at	17 ( 4 m )	,
30	.0087.3	114.480			ora prife	23.19.17		TO STAN	
31	.00002	110.802	202048 202099	37.4570	014 14 K	33.7514	100	170 100 1 8	+ 4
32 33	.00000	104.171	012700	Mantes	314 11 4	22.1543	1 1	67 1 Tests	4
34	.00080	101.107	July 15	36 26127	JAMES T	22.8 33	-3 2 . 1	1 4	
35	81010.	98.2170	19.27/14	A11. 2 7761	11.64 5 8 8	33 30 3 1	F 17 8	4 - 191 4 -	0.0
36	.01047	94.4894	102791	As Newston	114541	22 317	W 24, 8	3 - 0-4	8 .
37 38	.01076	00.4633	103823 103841	45-4114	21645 (12.4 2145 (14.4)	us fefer e	F. 1.1	re +19	- 1
39	.01135	88.14 (0	312881	44.7141	314Feath	12 6- 11	4 14	11	
40	40110.	85.0308	1120111	44. 10078	AMPLE N. T.	25 8 8	11.3 3	1 49	1.27
41	.01103	81.8415	202030	44 11374	10.12	or Tatas	4.3	3	
42	.01222	81.8470	Some !	34 Ca135 1	1914 TAFE	11 39	1 1	4. 4 3	18
4.3	.01251	70 04 44	21213137	\$ 4 (1/11/2)	04745	31 77	4.14	3	
44 45	.01300	78.1.204	104 126	\$4 (94) \$	010	5 - 14 4 ( ) ) 5 - 1   1   1   1	(4) let	8.	
46	.01338	74.7701	orma I	414211	411	to Fey 1 de	8 4	2.	
47	.01307	73.130 /	#0 12 2 1 T	42 x x 254 x	114 11 1 3	2 - Fall	. 4.		
48	.01300	71.613,1	110 4 7 4 4	41 8 2 cm	17. 建門大学管	2 - 9 97 -	3 - 1 -	4.	
40 50	.01425	70.14.33 68.7501	100年第2日 100年第1日	11 4426	192	1 11.4		18 4	
51	.01484				\$14.214			4 24	
52	.01404	66,1005	14445 £	3 (6844	- Caragilli	John Strain		1	
53	.01542	64.8680	right .	2-1-2-2-4	1 1111	100 100	7	5 5	
54	.01571	testinitry	211417	E + 1-12fe	11. 667	211 1	+		
55	.01000	62,4002	101346 i	184 11 18 18	20 Tp	447 119	4	1	
56 57	.01620	61. (870)	114 4541	119 112 11	1 : 1	tra + f + fr		4 7 C	
57 58	.01687	Sunding	31444 F	20 1 12 1	3 - 2	Rey 1 1 1	* **	6.0	
50	.01716	58.2014	1,441	213 76 12 1	1.44.5	\$19 SAN 19	1	6 1 1	
60	.01746	67 2000	1111113	278 1. 150 g	: 47	211 12		× 6	
7	Con	17.	et a second						
- 1	Co-tan.i 89	TAN.	COLAN		11 6 4 90	1 46. 1	** 4	~ 5	
ı	Oth		- 88	٠.	47		196	,	

	40	٠	59	> 1	. 6	o 1	1 7	· ,	,
,	TAN.	Co-tan.		CO-TAN.		CO-TAN.	TAN.	CO-TAN.	,
	IAN.	COTAN							-
0	.06993	14.3007	.08749	11.4301	.10510	9.51436 9.48781	.12278	8.14435 8.12481	6 5
I	.07022	14.2411	.08778	11.3919	.10540	0.46141	.12338	8.10536	5
2	.07051	14.1821	.08837	11.3163	.10500	9.43515	.12367	8.08600	5
3	.07080	14.1235	.08866	11.2789	.10628	9.40904	.12397	8.06674	5
4 5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	5
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	5
	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	5
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	5
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	5
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574		1 -
II	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	4
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	4
13	.07373	13.5634	.09130	10.9529	.10893	9.18028 9.15554	.12602	7.87895	4
14	.07402	13.5098	.09159	10.9178	.10922	0.13093	.12722	7.86064	4
15	.07431	13.4566	.00218	10.8483	.10081	9.10646	.12751	7.84242	1 4
	.07461 .07490	13.4039	.09247	10.8139	.11011	0.08211	.12781	7.82428	14
17	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	4
19	07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	778825	4
20	-07578	13.1969	.09335	10.7119	. 11099	9.00983	.12869	7.77035	4
21	.07607	13.1461	.00365	10.6783	.11128	8.98598	.12899	7.75254	3
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	3
23	.07665	13.0458	.09423	8116.01	.11187	8.93867	.12958	7.71715	3
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	3
25	07724	12.9469	.09482	10.5462	.11246	8.89185	-13017	7.68208	13
26	•077 <u>5</u> 3	12.8981	.09511	10.5136	.11276	8.86862 8.84551	.13047 .13076	7.66466	3
27	.07782	12.8496	.09541	10.4813	.11305	8.82252	.13106	7.63005	3
28	.07812	12.8014	.09570	10.4491	.11335	8.79964	.13136	7.61287	1 3
29 30	.07841 .07870	12.7536	.09629	10.3854	.11304	8.77689	.13165	7-59575	1 3
-		1	.00658	10.3538	.11423	8.75425	.13195	7.57872	1 2
31 32	.07899 .07929	12.6591	.09688	10.3224	.11452	8.73172	.13224	7.56176	1 2
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	2
34	.07987	12.5100	.09746	10.2602	.11511	8.68701	.13284	7.52806	2
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	2
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	-13343	7-49465	2
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	2
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	2
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432 .13461	7.42871	2
40	1				11		. (	7.41240	1
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.39616	1
42 43	.08221	12.1632	.09981	0.0187	.11747 .11777	8.51259	.13550	7.37999	1 3
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	I
45	-08309	12.0346	.10069	9.93101	.11836	8.44896	.13600	7.34786	1
46	.08339	11.9923	.10000	9.20211	.11865	8.42795	.13639	7.33100	1
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	1
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	1
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	1
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	3
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	1
52	.08514	11.7448	.10275	9.73217	.12042	8.30406 8.28376	.13817	7.23754	
53 54	.08544	11.7045	.10305	9.70441 9.67680	.12072	8.26355	.13846	7.22204	1
54	.08602	11.6248	.10334	9.64935	.12131	8.24345	.13070	7.19125	
55 56	.08632	11.5853	.10303	0.62205	.12131	8.22344	.13935	7.17594	1
57	.08661	11.5461	.10422	9.59490	.12100	8.20352	.13965	7.16071	1
58	-08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	
	***************************************								1
59	.08720	11.4685	.10481	9.54100	.12249	8.16398	.14024	7.13042	ł
59 60	.08720	11.4685	.10481	9.54106	.12249	8.10398	.14024	7.13042	
59 60 -	.08720	11.4301		9.51436		8.14435			-

	1 8	0	11 9	0	11 1	.00	li <b>1</b>	1°	1
,	TAN.	CO-TAN.	TAN.	Co-TAN.	TAN.	CO-TAN.		Co-tan.	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.06165	.19468	5.13658	59
2	.14113	7.08546	.15028	6.29007	.17693	5.65205	.19498	5.12862	58
3 4	.14143	7.07050	.15958	6.26655	.17723	5.64248	.19529	5.12069	52
	.14202	7.04105	.15988	6.25486	.17753 .17783	5.62344	.19559	5.11279	56
5 6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19519	5.00704	55 54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08021	53
	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.90823	.16137	6.19703	-17933	5.57638	.19740	5.06584	50
II	.14381	6.95385	.16167	6.18559	.17963	5.56706	19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	-17993 -		.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14499	6.89688	.16286	6.14023	.18083	5.53927 5.53007	19861	5.03499	46
16	1 .14529	6.88278	.16316	6.12899	.18113	5.52090	.19091	5.02734	45 44
17	.14550	6.80874	.16346	6.11779	.18143	5-51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10064	.18173	5.50264	.19982	5.00451	42
19	.14018	6.84082	.16405	6.00552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16405	6.06240	.18293	5.46648	.20103	4.97438	38
23 24	.14737	6.78504	.10525	6.05143	.18323	5.45751	.20133	4.96690	37
25	.14796	6.75838	.16555	6.02062	.18353 .18383	5.44857 5.43966	.20104	4.95945	36
20	.14826	6.74483	.16015	6.01878	.18414	5.43077	.20224	4.95201	35 34
27	14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14880	6.71780	.16674	5.00720	18474	5.41300	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.02249	31
30	-14945	0.69116	.10734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14075	6.67787	.16764	5.96510	.18564	5.38677	-20376	4.90785	20
32	.15005	6.05144	.16824	5.95448	.18594	5.37805	-20406	4.90056	28
33 34	.15004	6.63831	.10854	5.93335	.18654	5.36936	-20436 -20466	4.89330	27
35	15004	6.02523	.16884	5.92283	.18684	5.35206	.20400	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	-20527	4.87162	24
37	.15153	0.50021	.10944	5.00101	.18745	5.33487	-20557	4.86444	23
38	.15183	6.58627	.10074	5.80151	.18775	5.32631	-20588	4.85727	22
39 40	.15213	0.57339	.17004	5.88114	.18805	5.31778	-20018	4.85013	21
		6.50055	.17033		.18835	5.30928	420648	4.84300	20
41	.15272	0.54777	.17063	5.86051 5.85024	.18865	5.30080	-20679	4.83500	10
43	.15332	6.52234	.17003	5.84001	.18025	5.20235	-20709 -20739	4.82882	18
44	.15302	6.50070	.17153	5.82982	.18055	5.27553	-20770	4.81471	16
45	.15391	0.40710	.17183	5.81966	.18986	5.20715	.20800	4.80760	15
40	.15421	0.48450	.17213	5.80053	.19016	5.25880	-20830	4.80068	14
47	.15451	6.47206	.172.43	5.79044	.19046	5.25048	-20861	4.70370	13
48 49	.15481	6.45961	.17273	5.78038	.10076	5.24218	-20801	4.78073	12
50	.15540	6.43484	.17303	5.77036	.19136	5.23391 5.22566	.20021	4.77978	11
51	.15570	6.42253	.17363	5.75041	.19166	1	.20082		
52	.15000	0.41030	.17393	5.74949	.10107	5.21744	.21013	4.76595	8
53	.15030	6.30804	.17423	5.73000	.19227	5.20107	.21043	4.75219	7
54	.15000	6.38587	-17453	5.72974	-19257	5.10203	.21073	4.74534	ò
5.5	.15080	0.37374	.17483	5.71002	10287	5.18480	-21104	4.73851	5
56 57	.15710	6.36165	.17513	5.71013	.10317	5-17671	-21134	4.73170	4
58	.15749 .15779	6.34961	.17543 .17573	5.70037	.19347	5.10803	.21104	4.72400	3
50	.15800	6.32566	.17003	5.08004	.19498	5.15250	.21105	4.71813	r
bo	.15838	6.31375	.17033	5.07128	.19438	5.14455	.21256	4.70463	ô
-	Comar	There	Camari	(Day)	Commence and		(1		,
	CO-TAN.	TAN.	CO-TAN.	O TAN.	CO-TAN.	OO TAN.	CO-TAN.	TAN.	ľ
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,	TAN.	CO-TAN.	TAN.	CO-TAN	TAN.	Co-TAN	TAN.	CO-TAN.	1
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
ī	.21286	4.69791	.23117	4.32573	.24964		.26826	3.72771	59
2	-21316	4.69121	.23148	4.32001	-24995	4.00086	.26857	3.72338	58
3	-21347	4.68452	.23179	4.31430	-25026	3.99592	.26888	3.71907	
4	-21377	4.67786	.23209	4.30860	.25056	3.99099	.26020	3.71476	55
4	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
5	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
	.21460	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
7	.21400	4.65138	-23332	4.28595	.25180	3.97139	27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	-23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21500	4.63171	-23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	-23455	4.26352	.25304	3.95196	.27160	3.68061	48
13	.21651	4.61868	.23485	4.25795	-25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	-23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.50027	.23578	4.24132	.25428	3.93271	-27294	3.66376	44
17	.21773	4.59283	-23608	4.23580	-25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	-27357	3.65538	42
10	.21834	4.58001	.23670	4.22481	.25521	3.91839	-27388	3.65121	41
20	-21864	4.57363	.23700	4.21933	.25552	3.91364	27419	3.64705	40
21	-21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	-21025	4.56001	.23762	4.20842	.25614	3.90417	27482	3.63874	38
23	.21956	4.55458	-23793	4.20208	-25645	3.89945	-27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	-27545	3.63048	36
25	-22017	4.54196	.23854	4.19215	-25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23016	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
20	.22130	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	-22200	4.50451	.24039	4.15997	.25803	3.86208	.27764	3.60181	20
32	.2223I	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4-49215	.24100	4.14934		3.85284	.27826	3.59370	27 26
34	.22202	4.48600	.24131	4-14405	.25955	3.84824	.27858	3.58966	
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27020	3.58160	24
37 38		4:46764	.24223	4.12825	.26079	3.83449	-27952	3.57758	23
	.22414	4.46155	-24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	-22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	-22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	-22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	10
42	.22536	4.43735	-24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	-22597	4-42534	-24439	4.00182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	-24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47 48	.22009	4.40745	•24532	4.07639	.26390	3.78931	.28206	3.53785	13
49	.22750	4.40152	-24562	4.07127	.26421	3.78485		3.53393	II
50	.22781	4.38969	•24593 •24624	4.06107	.26483	3.78040	.28329	3.53001 3.52609	10
	.22811		1		1	3.77595	1		
51 52	.22842	4.3838t	-24655	4.05599	.26515	3.77152	.28391	3.52219	8
53	.22872	4.37793	.24686 .24717	4.05002	.26546	3.76700	.28423	3.51829	
54	.22003	4.36623		4.04586	.26577	3.76268	.28486	3.51441	7 6
55	.22934	4.36040	-24747	4.04081	.26639	3.75828	.28517	3.51053 3.50666	
56	-22064	4.35459	.24778 .24800	4.03578	.26670	3.75388	.28549		5
57	-22995	4.34879	.24840	4.03075	.26701	3.74950 3.74512	.28580	3.50279 3.49894	4
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	ī
50	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	o
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. 1	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	,
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	0 .286		741 .3	0573	3.270	85 .3	2.102	3.0770	- 1 8   -344	13 2.0042	1 (1
	1 .287		359 3	0005	3.267		2524				
	2 .287			5637	3.204		2550			35 ( 2.50S)	
	3 .287			2000	3.200		2588				
				732	3.257		10.01				
	5 .288			764	3.2505		653 685	3.0025			
	7 .288	05 3.400		700	3.2.47		717	3.0505			
				828	3.2.138		7.10	3.05340			
	9 .280			Sto	3.240.		782	3.050.10			
	10 .2899	, , , , ,		891	3.2371	4 32	814	3.047.40	-3475	8 2.877 0	f,.,
	11 .2902			923	3.2338	1 32	8.46	3.04450	3470	1 2.874 , 1	40
	2 .200			055	3.230.		878	3.04154	11 .348	4 1 2.82 160	48
	13 .2908		.    0	987	3.2271		11(	3.03854	-348 0	2 2 36302	47
	5 .2014				3.2238.			3.03550	-3.17.70	2 2.500124	40
1	6 .2917				3.2205			3.03.00			45
	7 .2021				3.2130			3.02003			4.4
	8 .2924	2 3.410			3.2100			3.02372			43
	9 .2927		0.1 .31		3.2073			3.02077	3505.		41
	0 .2030			10	3.20400			3.01783	-35000	2.89024	3.0
2	955			42	3.20070			3.01480	.35117		10
2.			. 11 0		3.10752			3.01100	-35150		34
2,					3.10420			3.00003	34183	281220	17
2					3-10100			3.00011	.35210	2 / 10/14	30
20	.2949				3.18775 3.18451	-3,3-		3.00310	+35748		.45
2 7	29520			3.1	3.18127	•333 •333		3,00028	-31,2%1	25440	.44
28	7.7.7		7 31.4		3.17804	3333		2.00447	-35444 -35446	2811.6	1.1
29					3.17481	-3.34		2.00158	.1.1.0	3 Sate, 4	11
30		100.00	. 11	30	3.17150	-3340	ю	2208868	.3 .412	2 5 2 301	3
31 32					3.16838	-3340	12	2.08(8)	-10145	2 821 11	20
3.3					3.10517	-335-	14 L	2.9820.1	- 15477	2016	يَّا فِيَ
34					3.10197	3355		2.08004	45,520	2 4161 1 1	27
35	-20780				3.15877 3.15558	+3358		2.07717	+45544.E	2517 1	211
30	.20811	3.3544.		, ,	3+15240	-3302 -3305		2.07430	42.6	in parane ;	2%
37	-20843	3.3508	7 .3179		3-140-2	-3308		2.06848	45641	2 30 45 1 1	24
38	.20875	3.3473		0 :	3.1.4605	-3371		2.00573	1.071	2 30 14 1 1 2 30 14 16 1	24
39 40	.20006 .20038	3-34377			3-142-88	-3375	1 2	2.06288	11.7 1	19.	21
41	1	3-3-102	11	1 '	3.13072	3378		2.900 03.1	+ 1574 1	4 , 49% 12	24
42	.30001	3-33070			3.13656	-3,3810		505724	48557	2 705.45	10
4.3	-300.33	3.32905			-13341	-3 (5 )		'-0 d 37	1700 4	2 , 19 " 11 1	14
4.1	-30005	3.32614			13027	-3 (88)		505155	1. 1/2	2.0 .1	17
45	.30007	3-12-04		, .,	124111	3301		594595	1.0.1		1/1
40	-30128	3-31014	-3204	3 3	.1 2087	3 1077			0.01		1.5
47 48	30100	3.31505	3207.		.11779	34 110		010.3	3 46		1 1
49	30102	3.31216	11 "		11104	-34/143		0 17 13 1	10 1		1 1
50	.30255	3-39521	-32130		11173	34975	1		l		11
51	.30287	3-30174	11	10	.108.12	-34195	)		A 6 . 1		
52	30310	3.20820	3220		10532	34140	12	0.010	1611- 2	270 3	4)
53	-30351	3-20484	32.67	1 .,	00014	-34473	1 .	alle, to	101.1	2 ,84, 4	4
54	-30382	3.20130	32200		00000	-3420 j		911		2 / 100	2
55	30414	3.28709	-32331		00208	-34230 -34230		01700	dition	2 , 112 1	C)
50	30440	3.28492	-32363	3.	oSooi	31703		915 3 1	41.41	~ , * + p+ p* +	3
57 58	-30478 -30500	3.28100	-32300		08685	31115			7 7.1	2.77.49	1
50	30541	3-27767	-32428		08370	-34 363	12	0.071	30	2. 40	3
60	30573	3-27085	-32400 -32492		08073	41110	1	es dure	A Car	1100	î
-	ne e) ar charge	ordensessa to the survivance.	.2.40%	1 .5.0	07768	-31133	1-	0.4.1	fo 49 1	4.1.1	i)
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0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60 59
I	.36430	2.74499 2.74251	.38420 .38453	2.60057	.40470	2.47095	.42516	2.35205	58
2	.36463 .36496	2.74004	.38487	2.59831	.40504	2.46888	.4255I	2.35015	57
3 4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73500	.38553 .38587	2.59381	.40572 .40606	2.46476	.42619	2.34636	55 54
6	.36595 .36628	2.73263	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
7 8	.36661	2.72771	-38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	-40707	2.45055	-42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.4279I	2.33693	50
11	.36760	2.72036	-38754	2.58038	.40775	2.45246	.42826 .42860	2.33505	49 48
12	.36793 .36826	2.71792	.38787	2.57593	.40843	2.44839	.42894	2.33130	47
13	.36859	2.71305	.38854	2.57371	.40877	2.44636	-42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40011	2.44433	-42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945 .40079	2.44230	.42998	2.32570	44
17 18	.36958 .36991	2.70577	•38955 •38988	2.56487	.41013	2.43825	.43067	2.32197	42
10	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	-37057	2.69853	-39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	-41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	-30122	2.55608	.41149 .41183	2.43019	-43205 -43239	2.31456	38 37
23	.37157 .37190	2.69131	.39156	2.55170	.41217	2.42618	.43274	2.31086	36
24 25	.37223	2.68653	-39223	2.54952	-41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	-39257	2.54734	.41285	2.42218	·43343	2.30718	34
27	.37289	2.58175	-39290	2.54516	.41319 .41353	2.42019	.43378 .43412	2.30534	33
28	-37322 -37355	2.67937	-39324 -39357	2.54299	.41387	2.41620	-43447	2.30167	31
29 30	.37388	2.67462	-39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	-39425	2.53648	-41455	2.41223	.43516	2.29801	29
32	·37455 ·37488	2.66989	-39458	2.53432	.41490	2.41025	-43550	2.20619	28
33	.37488	2.66752	-39492 -39526	2.53217	.41524 .41558	2.40827	-43585 -43620	2.29437	27
34 35	.37521	2.66281	-39559	2.52786	-41592	2.40432	-43654	2.29073	25
36	·37554 ·37588	2.66046	-39593	2.52571	-41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	-39626	2.52357	.41660 .41604	2.40038	-43724 -43758	2.28710	23
38	.37654 .37687	2.65576 2.65342	.39660 .39694	2.52142	.41728	2.39645	·43793	2.28348	21
39 40	.37720	2.65109	-39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	-37754	2.64875	.3976r	2.51502	-41797	2.39253	.43862	2.27987	10
42	.37787	2.64642	-39795	2.51280	.41831	2.39058	-43897	2.27806	18
43	.37820	2.64410	-39829 -39862	2.51076	.41865	2.38862	-43932 -43966	2.27626	17
44	-37853 -37887	2.64177	.39896	2.50652	-41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	-39930	2.50440	.41968	2.38279	.44036	2.27088	14
47 48	-37953	2.63483	.39963	2.50220	.42002	2.38084	-4407I	2.26909	13
	.37986	2.63252	·39997 ·40031	2.50018	.42036 .42070	2.37891	.44105	2.26730	II
49 50	.38053	2.62791	.40065	2.49597	.42105	2.37504	-44175	2.26374	10
51	.38086	2.62561	.40008	2.40386	.42130	2.37311	.44210	2.26196	9
52	.38120	2.62332	-40132	2.49177	.42173	2.37118	-44244	2.26018	8
53	.38153	2.62103	.40166	2.48967 2.48758	-42207	2.36925	-44279	2.25840	7
54	.38186	2.61646	.40200	2.48549	.42242	2.36541	·44314 ·44349	2.25486	5
55 56	.38253 .38286	2.61418	-40267	2.48340	.42310	2.36349	-44384	2.25309	4
57 58	.38286	2.61190	.40301	2.48132	-42345	2.36158	-44418	2.25132	3
58	.38320	2.60963	.40335 .40369	2.47924	.42379 .42413	2.35967 2.35776	•44453 •44488	2.24956	2 1
59 60	·38353 ·38386	2.60509	.40403	2.47500	.42447	2.35585	.44523	2.24/604	ô
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2.24604	.46631 .46666	2.14451	.48773 .48809	2.05030 2.04879	-50953	1.96261	60
2.2.1252	.40702	2.14125	.48845	2.04728	.50989 .51020	1.96120	59
2.2.1077	-40737	2.13003	.48881	2.04720	.51063	1.95979	58
2.23002	.40772	2.13801	.48017	2.04426	.51003	1.95038	57 56
2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
2.23553	.40843	2.13477	.48989	2.04125	.51173	1.95417	54
2.23378	-40879	2.13316	.40026	2.03075	51200	1.95277	53
2.23204	40014	2.13154	-49002	2.03825	.51246	1.95137	52
2.23030	.40050	2.12003	.40008	2.03075	.51283	1.94997	5 T
2.22857	.40985	2.12832	-49134	2.03526	-51310	1.94858	50
2,22683	.47021	2.12671	.40170	2.03376	.51356	1.94718	49
2.22510	-47050	2.12511	.40200	2.03227	-51393	1.94579	48
2.22337	.17002	2.12350	.40242	2.03078	.51430	1.94440	47
2.22104	.47128	2.12100	40278	2.02020	.51467	1.94301	46
2.21002	.47163	2.12030	.40315	2.02780	.51503	1.04102	45
2.21810	-47199	2.11871	-4035I	2.02031	.51540	1.04023	44
2.21047	-47234	2.11711	-49387	2.02483	-51577	1.93885	43
2.21475	17270	2.11552	-40423	2.02335	.51614	1.03746	42
2.21304	-47305	2.11392	-49459	2.02187	·51651	1.93008	41
2.21132	-47341	2.11233	-49495	2.02030	.51688	1.93470	40
2.20061	-47377	2.11075	-49532	2.018gr	-51724	1.93332	39
2.20700	-47412	2.10016	.40508	2.017.13	-51761	1.03195	38
2.20010	-47448	2.10758	40004	2.01500	-51798	1.93057	37
2.20440	-47483	2.10000	-40040	2.01449	-51835	1.92920	36
2.20278	-47510	2.10442	-49077	2.01302	.51872	1.92782	35
2.20108	-47555	2.10284	-10713	2.01155	.51900	1.92645	34
2.10038	-47500	2.10120	-40740	2.01008	.51046	1.92508	3.3
2.10700	47020	2.00000	.40780	2.00802	.51983	1.02371	32
2.10500	-47002	2.00811	-40822	2.00715	-52020	1.02235	31
2.10430	-47098	2.09054	.49858	2.00560	-52057	1.92098	30
2.10261	-47733	2.00408	-49894	2.00423	-52004	1.01062	20
2.10003	-47700	2.00341	-49931	2.00277	-52131	1.01820	28
2.18023	-47805	2.00184	-40007	2.00131	-52108	1.91000	27
2.18755	-478.10	2.00028	.5000.4	1.00086	.52205	1.01554	26
2.18587	-47870	2.08872	.500.10	1,00841	-522.12	1.01418	25
2.18410	47013	2.08500	.50070	1.00005	-52270	1.01282	24
2.18084	.47048 .47084	2.08405	.50113	1.00,550	-52316 -52353	1.011.17	23
2.17010	48.210	2.08250	.50185	1.00201	.52,300	1.00876	21
2.17740	.48055	2.08004	.50222	1.00110	.52427	1.007.11	20
				1.08072			
2.17582	.48001	2.07030	.50258	1.08828	-52404	1.00007	18
2.17410	.48163	2.07785	.50205 .50331	1.08084	.52501 .52538	1.00472	17
2.17083	.48198	2.07470	-50,368	1.085.10	-5-575	1.00203	16
2.10017	.48234	2.07321	50404	1.08300	.52613	1.000(0)	15
2.10751	.48270	2.07107	.504.41	1.08253	.52050	1.80035	1.4
2.16585	.48300	2.0701.1	50477	1.08110	-5.087	1.80801	1,3
2.16426	.48342	2.00800	.50514	1.07000	-52724	1.80667	12
2.16298	-48378	2.00700	.50550	1.07823	.52761	1.80533	II
2.100000	.48414	2.00553	.50587	1.07080	.52708	1.80400	10
2.15025	18.150	2.06.100	.50023	1.07538	.52836	1.80266	9
2.15700	.48480	2,00247	.50000	1.07305	-52873	1.80133	8
2.15500	.48521	2.00004	.50000	1.07.153	.52010	1.80000	7
2.15432	.48557	2.05042	-507.3.3	1.07111	-52047	1.88867	Ó
80.21.9	-48503	2.05700	.50760	1.00000	.52084	1.887.34	5
2.15104	.48620	2.05637	.50800	1.00827	-53022	1.88602	4
2.14040	.48065	2.05485	.508.1,3	1.00685	-53050	1.88460	3
2.14777	.18701	2.05333	50870	1.005.4.4	.53000	1.88337	2
2.1401.4	-48737	2.05182	-50010	1.00.102	-53134	1.88205	1
2.14451	.48773	2.05030	.50053	1.06201	.53171	1.88073	0
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0	.53171 .53208	1.88073 1.87941 1.87809	·55431 ·55469 ·55507	1.80405 1.80281 1.80158	-57774	1.73205 1.73089 1.72973	.60086 .60126 .60165	1.66428 1.66318 1. 6209	59 58
2 3 4	.53246 .53283 .53320	1.87677	·55545 ·55583	1.80034	.57851	1.72857	.60205	1.66099	57
5 6	.53358 -53395	1.87415	.55621 .55659	1.79788	-57929 -57968	1.72625	.60234	1.65881	55 54
7 8	-53432 -53470	1.87152	-55697 -55736	1.79542	.58007	1.72303	.60364	1.65663	53 52
9 10	.53507 .53545	1.86891	·55774 ·55812	1.79296	.58085	1.72163	.60443	1.65337	5 I 50
11	.53582 .53620	1.86630	.55850	1.79051 1.78929 1.78807	.58162 .58201 .58240	1.71032	.60522 .60562 .60602	1.65228	49
13 14 15	.53657 .53694 .53732	1.86369	.55926 .55964 .56003	1.78685	.58240 .58279 .58318	1.71702 1.71588 1.71473	.60642	1.64903	47 46 45
16	.53769	1.85979	.56041	1.78441	.58357 .58396	1.71358	.60721	1.64687	44
18	.53844 .53882	1.85720	.56117	1.78198	.58435 .58474	1.71129	.60801	1.64471 1.64363	42 41
20 21	-53920 -53957	1.85462	.56194 .56232	1.77955	.58513	1.70901	.60881	1.64256	40 39
22 23	-53995 -54032	1.85204	.56270 .56300	1.77713	.58591	1.70673	.60960	1.64041	38 37
25	.54070 .54107	1.84946	.56347 .56385	1.77471	.58670	1.70446	.61040	1.63826	36 35
26 27 28	.54145 .54183 .54220	1.84689 1.84561 1.84433	.56424 .56462 .56500	1.77230 1.77110 1.76990	.58748 .58787 .58826	1.70219 1.70106 1.69992	.61120 .61160	1.63612 1.63505 1.63398	34 33 32
29 30	.54258 .54296	1.84305	.56539 .56577	1.76869	.58865 .58904	1.69879	.61240 .61280	1.63292	31 30
31 32	·54333 ·54371	1.84049	.56616 .56654	1.76630	.58944 .58983	1.69553	.61320 .61360	1.63079	29 28
33 34	.54409 .54446 .54484	1.83794 1.83667 1.83540	.56693 .56731 .56769	1.76390 1.76271 1.76151	.59022 .59061	1.69428 1.69316 1.69203	.61400 .61440	1.62866 1.62760 1.62654	27 26 25
35 36 37	.54522 .54560	1.83413	.56808	1.75013	.59101 .59140 .59179	1.69091	.61520	1.62548	24 23
38 39	•54597 •54635	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
40 41	-54673 -54711	1.82906	.56962	1.75556	-59297 -59336	1.68643	.61681 .61721	1.62125	20
42 43	-54748 -54786	1.82654	.57030 .57078	1.75319	-59376 -59415	1.68419	.61761	1.61014	18
44 45	-54824 -54862	1.82402	.57116 .57155	1.75082	•59454 •59494	1.68196	.61842 .61882	1.61703 1.61598	16 15
16 47 48	.54900 .54938 -54975	1.82150 1.82025 1.81899	·57193 ·57232	1.74846	·59533 ·59573	1.67974	.61962	1.61403	14
49	.55013 .55051	1.81774 1.81649	.57271 .57300 .57348	1.74610 1.74492 1.74375	.59612 .59651 .59691	1.67752 1.67641 1.67530	.62003 .62043 .62083	1.61283 1.61179 1.61074	12 11 10
51 52	.55089 -55127	1.81524	-57386 -57425	I.74257 I.74140	-59730	1.67419	.62124	1.60070	9
53 54	.55165 .55203	1.81274	.57464 .57503	1.74022	.59770 .59809 .59849	1.67300 1.67198 1.67088	.62164 .62204 .62245	1.60657	7
55 56	-5524I -55279	1.81025	·57541 ·57580	1.73788	.59888	1.66978	.62285	1.60553	5
57 58	-55317 -55355	1.80777	-57619 -57657	1.73555	.59967	1.66757	.62366	1.60345	3
59	·55393 ·55431	1.80529	.57696 .57735	I.7332I I.73205	.60046 .60086	1.66538	.62446 .62487	1.60137	I O
1	Со- <sub>тан</sub> . [ 6]	TAN.	Co-tan. 6(	TAN.	Co-tan.	TAN.	Co-tan.	TAN.	-

}	3	20	3	3°	3	40	3	5°	
,	TAN.	Co-tan.	TAN.	Co-tan.	TAN.	Co-tan.	TAN.	Co-tan.	_
0	62487	1.60033	.64941	1.53986	.67451	1.48256	-70021	1.42815	60
I	.62527	1.59930	.64982	1.53888	-67493	1.48163	-70064	1.42726	59 58
2	.62568 .62608	1.59826	.65023	1.53791	.67536 .67578	1.48070	.70107	1.42638	
3 4	.62649	1.59723 1.59620	.65106	1.53595	.67620	1.47885	-70194	1.42462	57 56
7	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
5	62730	1.50414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7 8	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
	.62811	1.59208	65272	1.53205	67790	1.47514	.70368	1.42110	52
10	.62852 .62892	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	50
		1.59002	.65355			1.47330			
11	.62933 .62973	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49 48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
10	.63217 .63258	1.58184	.65688	1.52235	.68215	1.46595	.70848	1.41235	42 41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	-70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	33
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34 33
28	.63625	1.57271	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57060	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	-71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830 .63871	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27 26
34 35	.63912	1.56566	.66356 .66398	1.50607	.68942	1.45139	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	-71593	1.39679	24
37	.63094	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.04035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725 .71769	1.39421	2I 20
40	.64117	1.55966		1.50133	.69157	1.44598	.71813	1.39336	
41 42	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71857	1.39250	19
43	.64240	1.55666	.66734	1.49849	.60286	1.44329	.71001	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45 46	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
	.64363	1.55368	.66860	1.49566	.69416	1.44060	-72034	1.38824	14
47 48	.64404	1.55269	.66902 .66944	1.49472	.69459	1.43970	.72078	1.38738	13
49	.64487	1.55170	.66986	1.49378	.69545	1.43792	.72166	1.38568	II
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.6063T	1.43614	.72255	1.38399	n
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7 6
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	5
55 56	.64775	1.54478	.67239	1.48722	.60847	1.43258	.72432	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
57 58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42003	.72610	1.37722	I
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72054	1.37638	_ °
,	CO-TAN	TAN.	Co-TAN.	TAN.	Co-TAN.	TAN.	CO-TAN.	TAN.	,
	l'o lan	70	July	60	F	50	5	40	1
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,	TAN.	CO-TAN.	TAN.	Co-tan.	TAN.	CO-TAN.	TAN.	Co-TAN.	_
0	.72654	1.37638	·75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.7540I	1.32624	-78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	·75492	1.32464	.78269	1.27764	.81123 .81171	1.23270	57 56
4	.72832	1.37302	.75538	1.32384	.78316	1.27688		1.23123	
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220 .81268	1.23050	55 54
6	.72921	1-37134	.75629	1.32224	.78410	1.27535	.81316	1.22077	53
7	.72966	1.37050	.75675	1.32144	-78457	1.27458	.81364	1.22004	52
8	.73010	1.36967	-75721	1.32064	.78504	1.27382 1.27306	.81304	1.22831	51
9	-73055	1.36883	.75767	1.31984	.78551 .78598	1.27230	.81461	1.22758	50
13	-73100	1.36800	.75812	1.31904			.81510	1.22685	40
II	-73144	1.36716	.75858	1.31825	.78645	1.27153	.81518	1.22012	48
12	.73189	1.36633	-75904	1.31745	.78692	1.27077	.81606	1.22530	47
13	-73234	1.36549	-75950	1.31666	-78739	1.27001	.81655	1.22407	40
14	-73278	1.36466	-75996	1.31586	.78786	1.26840	.81703	1.22304	45
15	·73323	1.36383	.76042	1.31507	.78834 .78881	1.26774	.81752	1.22321	44
16	.73368	1.36300	.76088	1.31427	.78928	1.26698	.81800	1.22249	43
17	.73413	1.36217	.76134	1.31348	.78975	1.26622	.81849	1.22170	42
18	-73457	1.36133	.76180	1.31269	.79022	1.26546	.81898	1.22104	41
19	-73502	1.36051	.76226 .76272	1.31110	.79070	1.26471	.81946	1.22031	40
20	•73547	1.35968			.79117	1.26305	.81995	1.21050	30
21	73592	1.35885	.76318	1.31031	.79164	1.26319	.82044	1.21886	38
22	•73637	1.35802	.76364	1.30873	.79104	1.26244	.82002	1.21814	37
23	-7368I	1.35719	.76456	1.30795	-79259	1.26169	.82141	1.21742	30
24	-73726	1.35637	.76502	1.30716	.79306	1.26003	.82190	1.21070	35
25 26	-73771 -73816	1.35472	.76548	1.30637	•79354	1.26018	.82238	1.21508	34
27	-73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21520	33
28	-73906	1.35307	.76640	1.30480	-79449	1.25067	.82336	1.21454	3.2
20	·73951	1.35224	.76686	1.30401	-79496	1. 25702	.82385	1.21382	31
30	-73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	-7404I	1.35060	.76779	1.30244	-7959I	1.25642	82483	1.212,38	20
32	-74086	1.34978	.76825	1.30166	-79039	1.25567	.82531	1.21100	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.2100.4	27
34	•74176	1.34814	.76918	1.30009	-79734	1.25417	.82020	1.21023	20
35	·7422I	1.34732	.76964	1.29931	.7978r	1.25343	.82678	1.20051	25
36	-74267	1.34650	.77010	1.29853	.79829	1.25208	.82727	1.20879	2.3
37	-74312	1.34568	77057	1.29775	.79877	1.25193	.82776 .82825	1.20736	22
38	•74357	1.34487	.77103	1.29696	-79924	1.25118	.82874	1.20005	21
39	-74402	1.34405	-77149	1.29618	.79972 .80020	1.25044	.82923	1.20593	20
40	-74447	1.34323	.77196	1.20541	1				10
41	-74492	1.34242	.77242	1.29463	.80067	1.24805	.82072	1.20522	18
42	-74538	1.34160	.77289	1.29385	.80115	1.24820	.83071	1.20370	17
43	.74583 .74628	1.34079	·77335 ·77382	1.20307	.80211	1.24/40	.83120	1.20308	10
44	.74674	1.33998	.77428	1.20229	.80258	1.24597	.83160	1.20237	15
45 46	.74719	1.33835	-77475	1.20074	.80306	1.24523	.83218	1.20100	1.4
47	.74764	1.33754	.77521	1.28007	.80354	1.24449	.83268	1.20005	1.3
48	.74810	1.33673	.77568	1.28010	.80402	1.24375	.83317	1.20024	1.2
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83306	1.10053	11
50	•74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	-74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	0
52	·74991	1.33349	-77754	1.28610	.80594	1.24079	.83514	1.10740	8
53	.75037	1.33268	-77801	1.28533	.80642	1.24005	83564	1.10000	7
54	.75082	1.33187	.77848	1.28456	.80600	1.23031	.83613	1.10500	()
55	.75128	1.33107	.77895	1.28379	-80738	1.23858	.83662	1.10528	- 5
56	-75173	1.33026	·77941	1.28302	.80786	1.23784	.83712	1.10457	-4
57 58	.75219	1.32946	.77988	1.28225	.80834 .80882	1.23710	.83761	1.10387	3
59	.75264 .75310	1.32785	.78035	1.28071	.80030	1.23637	.83860	1.10246	1
60	·75355	1.32704	.78129	1.27004	.80938	1.23503	.83010	1.10175	ò
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•	CO-TAN.	TAN.	Co-tan.	TAN.	Co-tan.	TAN.	Co-tan.	TAN 0°	

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	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	Co-TAN.	1
	.83010	1.10175	.86929	1.15037	.00040	1.11061	.93252	1.07237	60
0	.83960	1.19175	.86980	1.14000	.90093	1.10006	-93306	1.07174	59
I	.84000	1.10035	.87031	1.14002	.90146	1.10031	.93360	1.07112	58
2	.84050	1.18064	.87082	1.14834	.90199	1.10867	-93415	1.07049	57
3	.84108	1.18894	.87133	1.14767	.90251	1.10802	-93469	1.06087	50
4	.84158	1.18824	.87184	1.14600	.00304	1.10737	-93524	1.00925	55
5	.84208	1.18754	.87236	1.14632	.90357	1.10072	.93578	1.06862	54
	.84258	1.18684	.87287	1.14565	.90410	1.10007	.93633	1.06800	53
7 8	.84307	1.18614	.87338	1.14498	.90.163	1.10543	.93688	1.06738	52
	.84357	1.18544	.87380	1.14430	.90516	1.10478	-93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.00500	1.10414	-93797	1.00613	50
- 1		1.18404	.87492	1.14206	.00621	1.10340	.03852	1.06551	49
II	-84457	1.18334	.87543	1.14220	.00674	1.10285	.93906	1.06480	48
12	.84507 .84556	1.18264	.87505	1.14162	.90727	1.10220	.93961	1.06427	47
13	.84606	1.18194	.87646	1.14005	.00781	1.10156	.94016	1.06365	40
14	.84656	1.18125	.87698	1.14028	.90834	1.10001	.94071	1.06303	45
15	.84706	1.18055	.87749	1.13061	.00887	1.10027	.94125	1.06241	44
17	.84756	1.17086	.87801	1.13894	.00040	1.00063	.94180	1.06179	43
18	.84806	1.17016	.87852	1.13828	.00003	1.00800	.94235	1.06117	42
10	.84856	1.17846	.87004	1.13761	.010.46	1.00834	.04290	1.00056	41
20	.84006	1.17777	.87955	1.13094	.01000	1.09770	-94345	1.05994	40
21	*84956	1.17708	.88007	1.13627	.01153	1.00706	.94400	1.05032	39
21	.85006	1.17038	.88050	1.13561	.01200	1.00042	-94455	1.05870	38
23	.85057	1.17509	.88110	1.13494	.91250	1.00578	-04510	1.05800	37
24	.85107	1.17500	.88102	1.13428	91313	1.00514	-94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.01300	1.00.150	.94620	1.05685	35
26	.85207	1.17301	.88265	1.13205	.01410	1.00386	.94676	1.05624	34
27	.85257	1.17202	.88317	1.13228	.01473	1.00322	9473I	1.05562	33
28	.85307	1.17223	.88300	1.13162	01520	1.00258	-04786	1.05501	32
20	.85358	1.17154	.88421	1.13006	.91580	1.00195	-0484 r	1.05430	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	<b>.</b> 94896	1.05378	30
3.1	.85458	1.17016	.88524	1.12063	.01687	1.00067	.04052	1.05317	20
32	85500	1.10047	.88576	1.12807	.01740	1.00,003	.05007	1.05255	28
33	85550	1.16878	.886.28	1.12831	.01704	04080.r	.05002	1.05194	27
34	.85000	1.10800	.88686	1.12705	.018.47	1.08876	.95118	1.05133	20
3.5	.85000	1.10741	.887.32	1.12000	10010	1.08813	-95173	1.05072	25
36	.85710	1.16072	.8878.4	1.12033	-01055	1.08740	.05220	1.05010	2.4
₹7	.85761	1.10003	.888,36	1.12507	.02008	1.08080	.05284	1.04040	23
38	.85811	1.10535	.88888	1.12501	.02002	1.08022	-05340	1.04888	21
30	.85862	1.16466	.880.40	1.12435	.02116	1.08550	-05305	1.04700	20
40	.85912	1.16398	.88902	1.12300	.02170	1.08490	-95451		1
1 r	.85063	1.16320	.80045	1.12303	.02224	1.08432	-05506	1.04705	10
42	.86014	1.16261	.80007	1.12238	.02277	1.08360	05502	1.04644	17
4.3	4,0008.	1.10102	.801.40	1.12172	.02331	1.08,300	.05618	1.04583	16
4.4	.86115	1.10124	80501	1.12100	-02385	1.08243	-05073	1.04401	15
4.5	.86166	1.10050	80253	1.12041	-92439	1.08170	.05720	1.04401	14
40	86216	1.15087	.80300	1.11075	-02403	1.08053	05811	1.0.13.10	13
47	.86267	1.15010	.80358	1.11000	.02547	1.07000	.05807	1.04270	12
48	.86318	1.15851	80410	1.11844	.02055	1.07027	-05052	1.04218	11
40	.86368	1.15783	.80463	1.11713	.02700	1.07864	.00008	1.04158	10
50	.86419	1.15715	11	1	11	1.07801	1,00004	1.0.1007	0
51	.86470	1.15647	.80507	1.11648	.02763	1.077.38	05130	1.04030	8
52	.86521	1.15570	.80620	1.11502	02872	1.07070	.00170	1.03070	7
53	86572	1.15511	80725	1.11452	.02026	1.07013	.00232	1.03015	0
5.1	86623	1.15.143	80777	1.11387	.02080	1.07550	.96288	1.03855	5
55 56	.86074	1.15375	.80830	1.11321	.03033	1.07.187	.00344	1.03794	4
	86776	1.15240	.80883	1.11250	.03088	1.07425	.00,100	1.03734	3
57 58	.86827	1.15172	.899.35	1.11101	.031.43	1.07302	-00457	1.03074	2
50	.86878	1.15104	.80088	1.11120	.03107	1.07290	.00513	1.03613	1
00	.86929	1.15037	.000.10	1.11061	.03252	1.07237	.00500	1.03553	0
•				-	-		1.	70	-
,	CO-TAN	TAN.	CO-TAN	TAN.	CO-TAN	TAN.	CO-TAN	il Tan 460	1
	1 .	190	11 4	18°	II .	170	11	30	1

	1 4	<b>4</b> °	I	11	1 4	4°	1 1	1	1 4	<b>[4</b> °	ĺ
,	TAN.	Co-tan.	1	′_	TAN.	Co-TAN.		'	TAN.	Co-tan.	′
0	.96569	1.03553	60	2I 22	.97756	1.02295	39 38	41	.98901 .98958	1.01112	19
1 2	.96625 .96681	1.03493	59 58	23	.97870	1.02176	37	42	.99016	1.00094	17
3	.96738	I.03372	57	24	-97927	1.02117	36	44	-99073	1.00935	16
4	-96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	1.4
	.96907	1.03192	54 53	27	.98155	1.01879	33	48	•99247 •99304	1.00759	12
7	.97020	1.03072	52	20	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	-99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	20	51	-99478	1.00525	9
II	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	-97246	1.02832	48	33	.98441	1.01583	27 26	53	-99594	1.00408	7
13	-97302	1.02772	47 46	34	.98499	1.01524	25	54 55	.99652 .99710	1.00350	5
15	-97359 -97416	1.02713	45	35 36	.98613	1.01406	24	56	.99768	1.00231	4
16	-97472	1.02503	44	37	.08671	1.01347	23	57	.99826	1.00175	3
17	-97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	-97643	1.02414	41	40	.98843	1.01170	20	65	I	1	٥
20	.97700	1.02355	40								
'	CO-TAN.	TAN.	'	′	CO-TAN.	TAN.	1	1	CO-TAN.	Tan.	′
	45°			i	4	J- {	1	1	4.	٠- ا	

6. Complete the following table using the table of sines cosines, etc.

From the above table what do you find about the sin  $30^{\circ}$  and the  $\cos 60^{\circ}$ , [that is, the  $\cos 60^{\circ} - 30^{\circ}$ ], also about the  $\sin 20^{\circ} 10'$  and the  $\cos 60^{\circ} - 20^{\circ} 10'$ ), that is, the  $\cos 69^{\circ} 50'$ ; about the  $\tan 12^{\circ}$  and the  $\cot 60^{\circ} - 12^{\circ}$ ) or  $\cot 78^{\circ}$ . In other words, this is true.

The sine of any angle less than 90° equals the co-function or cosine of (90° minus that angle), also, the tangent of any angle less than 90°, for example, the tangent of  $12^{\circ}$  equals the co-function or cotangent of  $(90^{\circ} - 12^{\circ})$ 

or equals the cot of 78°. In the same way the sine 20°  $10' = \cos .69^{\circ} 50'$ , the cot  $48^{\circ} = \tan (90^{\circ} - 48^{\circ})$  or  $\tan 42^{\circ}$  the  $\cos 30^{\circ} = \sin . (90^{\circ} - 30^{\circ})$  or  $\sin 60^{\circ}$ , etc. This is a good rule to remember.

62. Line Values of Functions.—The values of the sine.

cosine, tangent and cotangent of angles can be represented by the lengths of lines as follows: In Fig. 109 the circle with center O is drawn with a radius (one-half the diameter) equal to 1 in. CD is a horizontal and EF a vertical diameter. The radius OA makes any convenient angle. "a" with radius OC and from point A line

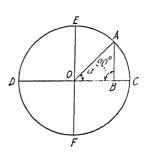


Fig. 109.

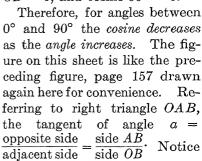
AB is drawn at right angles to line OC making the angle at B equal to 90° and giving the right triangle OAB. From the definition of the sine  $\left(-\frac{\text{opposite side}}{\text{hypothenuse}}\right)$ have in the right triangle OAB. The sine of angle side AB. But OA = 1 by construction (since we made the circle with OA = 1 in.). Therefore sine  $a = \operatorname{side} AB$ or side AB. Hence the sine of a is equal to the length of the line AB.—In the figure, AB measure .5 of an inch. Therefore sine a .5. Notice that if angle a were smaller the line AB would be smaller, that is, sine a would be smaller. If angle a were larger, line AB would be larger and therefore sine a would be larger. When angle  $a = 0^{\circ}$ , line AB = 0 in, and sine a = 0, that is, sine  $0^{\circ} = 0$ . When  $a = 90^{\circ}$ , line AB = 1me OE in length or 1 in, and therefore sine  $90^{\circ}$  = 1. There

fore for angle between  $0^{\circ}$  and  $90^{\circ}$  the sine *increases* when the angle *increases*.

Referring again to Fig. 109, in the right triangle OAB the cosine of angle  $a = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{\text{side }OB}{\text{hyp. }OA} = \frac{\text{side }OB}{1}$  or side OB.

That is, the line OB equals to scale the cosine of angle a. When angle a=0, OB=OC, and  $\cos$ 

a = 1. When angle  $a = 90^{\circ}$ , line OB = 0, and cosine  $90^{\circ} = 0$ .



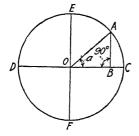


Fig. 110.

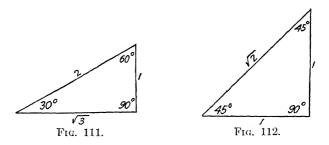
that as angle a increases, side AC (that is, the numerator of the fraction which is the tangent) increases, and at the same time side OB, the denominator of the fraction, decreases. Therefore as angle a increases the tangent increases and also if angle a decreases the tangent of angle a decreases.

In right triangle OAB the cotangent of angle  $a = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{\text{side }OB}{\text{side }AB}$ . Then as angle a increases, side OB decreases and side AB increases; therefore the fraction representing the cotangent decreases. As angle a decreases side OB increases and side AB decreases, and the value of the fraction increases, that is, the cotangent

increases. The following table for values of 0° and 90° angles should be carefully studied and understood from Fig. 110.

Angle	$\operatorname{Sine}$	Cosine	Tangent	Cotangent
$0^{\circ}$	0	1	0	Infinity
$90^{\circ}$	1	0	Infinity	0

63. Values of the Sine, Cosine, Tangent, and Cotangent for the Common Angles.—In a 30°-60° right



triangle shown in Fig. 111 the sides always have a ratio of 1, 2, and  $\sqrt{3}$  as given. You can prove this for yourselves by carefully constructing such a triangle on the drawing board and measuring the lengths of the sides. Therefore from our definition of the sine, cosine, etc., of an angle we have

Sin 
$$30^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{1}{2} = .5$$
  
Cos  $30^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{\sqrt{3}}{2} = .866$   
Tan  $30^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{1}{\sqrt{3}} = .577$ 

Cot 
$$30^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{\sqrt{3}}{1} = 1.732$$
  
Sin  $60^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{\sqrt{3}}{2} = .866$   
Cos  $60^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{1}{2} = .5$   
Tan  $60^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{\sqrt{3}}{1} = 1.732$   
Cot  $60^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{1}{\sqrt{3}} = .577$ 

In a 45°-45° right triangle as shown in Fig. 112 the sides always have a ratio of 1, 1 and  $\sqrt{2}$  as shown. Therefore for this figure:

Sin 
$$45^{\circ} = \frac{\text{opp. side}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = .707$$
  
Cos  $45^{\circ} = \frac{\text{adj. side}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = .707$   
Tan  $45^{\circ} = \frac{\text{opp. side}}{\text{adj. side}} = \frac{1}{1} = 1.00$   
Cot  $45^{\circ} = \frac{\text{adj. side}}{\text{opp. side}} = \frac{1}{1} = 1.00$ 

These values are tabulated below for reference.

Angle	Sine	Cosine	Tangent	Cotangent
30°	.5	.866	. 577	1.732
$45^{\circ}$	.707	.707	1.00	1.00
60°	.866	.5	1.732	. 577

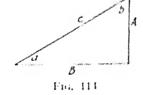
64. Methods of Working out Right Triangles.—In working out problems by the use of the sine, cosine, etc., we use one or the other, according to the sides of the triangle which are to be found and the sides which are

1111

or example in Fig. 113 the hyp. 10 in, and acute angle = 30°. We want to find side id angle b. In any right triangle the sum of ite angles equals 90°, therefore angle h +and angle  $b \approx 90^{\circ} - 30^{\circ}$  or  $60^{\circ}$ . To find ave sine  $30^{\circ}$  opp. side side A. hyp. 10 in.

 $\frac{4}{3}$ , side  $A = 10 \times \sin 30^{\circ}$  (sine 30 = -5).





Let  $A = 10 \times .5$  or 5 in. To find side B osine as follows:

adj. side - side B $\cos 30^{\circ}$ hyp. 10

le B = 10 Kilcos 30', teos 30', 866. B = 10 ≥ .866 or 8,66 in.

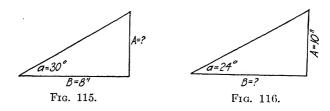
explanation just given and using Lip 114 · following table. The first set of value orked out as a guide.

100 to 100 to 100 to 10				
e a	Angle 6	selv k	5-4 <b>t</b>	1. 11
. •	60 '	16	8	1. "
		24		
	30 '	.; >		
		i a		
•		(+,2		
	75			

Draw a separate triangle for each set of values and put the lengths of sides and sizes of angles on each figure as well as putting them in the table.

If as in the triangle shown (Fig. 115) we had given angle  $a = 30^{\circ}$  and side B = 8, to find side A we use the tanget as follows:  $\tan 30^{\circ} = \frac{\text{side } A}{\text{side } B}$ , therefore side  $A = \text{side } B \times \tan 30^{\circ}$  or side  $A = 8 \times .577 = 4.62$ .

If as in the triangle shown in Fig. 116 we have the side



A=10 and angle  $a=24^{\circ}$  and wish to find side B we use the cotangent as follows:

$$\cot 24^{\circ} = \frac{\text{side } B}{10}$$

Therefore side  $B = 10 \times \cot 24^{\circ}$  or side  $B = 10 \times 2.25$  or 22.5. From the explanation just given and using Fig. 117 complete the following table.

-	Angle a	Angle $b$	Side A	Side $B$
1		29°	16	and the second s
<b>2</b>	80°			8.4
3		36°	35	
4	15°			45
5		75°	7.5	
6	60°			24

When the base B and angle a are given (Fig. 118) to find the hypothenuse proceed as follows:

$$\cos 40^{\circ} = \frac{\text{side } B}{\text{hyp.}}$$
$$\cos 40^{\circ} = \frac{20}{\text{hyp.}}$$

Fig. 117.

or

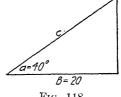
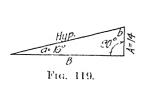


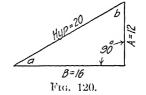
Fig. 118.

Therefore  $\cos 40^{\circ} \times \text{hyp.} = 20$  and from this the hyp. =

$$\frac{20}{\cos 40^{\circ}} = \frac{20}{.766} = 26.1$$

When the side A and angle a are given (Fig. 119) to find the hypothenuse proceed as follows:





 $\sin 16^{\circ} = \frac{\text{side } A}{\text{hyp.}} \text{ or } \sin 16^{\circ} = \frac{14}{\text{hyp.}}$ 

Therefore hyp.  $\times \sin 16^{\circ} = 14$  and hyp.  $= \frac{14}{\sin 16^{\circ}}$  or hyp. =  $\frac{14}{276}$  = 50.7

•	• ,	_		
Angle a	Angle $b$	Side A	Side B	
12°		10		
	16° 30′		20	
43°		15		
	27° 10′		63	
36°	000	23.8	40	
	32		42	
		Angle a Angle b  12° 16° 30′ 43° 27° 10′	Angle a Angle b Side A  12° 10 16° 30′ 43° 27° 10′ 36° 23.8	Angle $a$ Angle $b$ Side $A$ Side $B$ 12°     10     20       43°     15     63       36°     23.8

7. From the explanation just given fill in the following tal

When we have two sides of a right triangle and to find the angles we proceed as follows: In the tr shown in Fig. 120 A = 12, B = 16, hyp. = 20 find angles a and b.

Sin 
$$a = \frac{\text{opp. side}}{\text{hyp.}} = \frac{12}{20} = \frac{3}{5} = .6$$
. If sin  $a = .6$  angle  $a$  is one whose sine is .6. We therefore look tables for an angle whose sine is .6 and on page 14

the number .59995 which is the nearest one to .6 the angle is therefore 36° 52′.

We can find angle a also from the rule

$$\cos a = \frac{\text{adj. side}}{\text{hyp.}} = \frac{16}{20} = \frac{8}{10} = .8$$

Looking in the tables for the angle whose cosine is . we find on page 142 the angle 36° 52′.

We can find angle a also from the tangent r follows:

Tan 
$$a = \frac{\text{opp. side}}{\text{adj. side}} = \frac{12}{16} = \frac{3}{4} = .75$$

and therefore (page 154) angle  $a=36^{\circ}$  52', or fro cotangent rule

$$\cot a = \frac{16}{12} = \frac{4}{3} = 1.33$$

and from the tables (page 154) angle  $a = 36^{\circ} 52'$ .

Having obtained angle a from any one of the rules just given, angle b equals  $90^{\circ}$  – angle a, since angle a + angle  $b = 90^{\circ}$ .

8. From the explanation just given and using Fig. 121, find the value of the sine of angle a and the value of angle a, completing the following table.

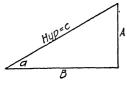


Fig. 121.

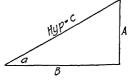


Fig. 122.

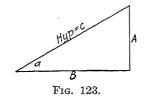
	Side A	Нур.	Sine a	Angle a
1	25	50		
2	7	35		
3	28	38		
4	15	20		
5	12	32		

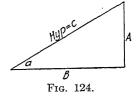
9. Using Fig. 122, find the value of the cosine of angle a and angle a, completing the following table.

	Side B	Hyp.	Cosine a	Angle a
1 2 3 4 5	10 3 30 16 14	20 4 75 28 32		an managang alah Promasa — Ser - S

10. For Fig. 123, find the tangent of angle a and the value of angle a, completing the following table.

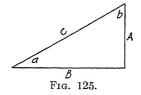
	Side A	Side B	Tangent a	Angle a
1 2 3 4	10 36 3 56	24 70 8 40		
5	38	112		





11. For Fig. 124, find the cotangent of angle a and the value of angle a, completing the following table.

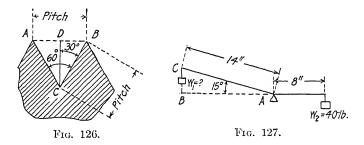
	Side A	Side $B$	Cotangent a	Angle a
1	7	32		
2	27	38		
3	14	18	1	
4	12	16		
5	23	42		



12. Using Fig. 125, complete the following table, drawing a separate triangle for each problem.

A	Angle α	Angle $b$	Side $A$	Side B	Side C
1 2 3 4 5 6	60° 75° 15°	40° 32°	15 20 25	35 30	75 50

- 65. Applications of Triangular Functions.—Following are applications of the preceding rules to the solution of problems in daily shop work.
- 13. To find the depth of a V thread. Fig. 126 shows a section of the thread. To find the depth of the thread or distance CD



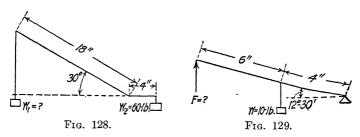
in triangle *DBC*. Side  $CD = \text{side } CB \times \cos 30^{\circ} = \text{pitch } \times .866$ . The pitch in in. always equals threads per in.

Find the depth of V threads as follows: 8 per in., 20 per in., 12 per in., and 40 per in.

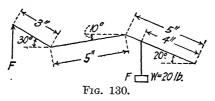
- 14. In a right triangle the hypothenuse is 20 in. and the height is to be twice the base. Find the height, base, and all the angles of the triangle. (Note.—Let the base be x in. and the height 2x in. The tangent of the base angle will then be  $\frac{2x}{x}$  or 2.)
  - 15. Fig. 127 shows a first-class lever. If  $W_2 = 40$  lb. to find

 $W_1$  we have the rule  $40 \times 8 = W_1 \times \text{distance } AB$ , but length  $AB = 14 \times \cos 15^\circ = 14 \times .966 = 13.5$ , therefore  $40 \times 8 = W_1 \times 13.5$  and  $W_1 = \frac{40 \times 8}{13.5} = 23.7 \text{ lb.}$  Ans.

16. With a first-class lever as in Fig. 128, find  $W_1$  necessary to balance  $W_2$  with lever arms as shown.

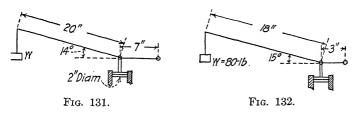


17. As per Fig. 129, find the force necessary at F to balance the weight of 10 lb. shown.

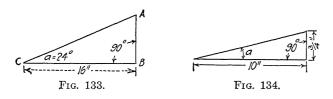


- 18. As shown in Fig. 130, find the force necessary at F to balance the 20 lb. What class of lever is this one?
- 19. What weight in lb. at W (Fig. 131) is required to set the safety valve so as to lift at 200 lb. per sq. in. pressure with a 2-in. diam. valve?
- 20. As per Fig. 132, what size valve is required with steam at 150 lb. per sq. in. to lift the weight of 80 lb. at W, with the lever arms as shown?
- 21. In a laying out job the distance CB Fig. 133 is taken as 16 in., the angle  $a=24^{\circ}$ . Find the distance between centers A and B.
- 22. The tangent of an angle is .45573. What are its sinc, cotangent, and cosine?

- 23. For the wedge shown (Fig. 134) find the angle between its sides.
- 24. Find the angle between the sides of a wedge 1 ft. long,  $\frac{1}{16}$  in. wide at one end and  $\frac{1}{16}$  in. wide at the other end.
- 25. The sides of a wrought-iron wedge form an angle of 2° 23'. Find the taper (decrease in thickness) per ft. of length.



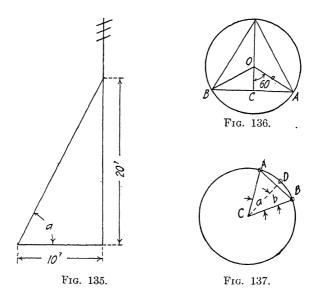
- 26. A reamer  $1\frac{5}{6}$  in. at the small end is 16 in. long and has a aper of  $\frac{3}{4}$  in. in. 12 in. ( $\frac{3}{4}$  in. increase in diam. per ft.). What is he diameter at the large end and what is the angle between the wo sides?
- 27. Find the angle which the sides of a wedge make with one nother for a taper of  $\frac{7}{8}$  in. per ft.



- 28. Fig. 135 shows a telegraph pole with a guy wire attached ) ft. up on pole and anchored 10 ft. from base of pole. Find ngth of wire required. Allow 2 ft. extra for fastening wire. Note.—Find first angle a, then get side C from the sine or cosine le.)
- 29. Find the angle of incline for a grade of 3 per cent., and the amber of feet rise in a distance of 2 miles. (A 1 per cent. grade one which rises 1 ft. vertically for every 100 ft. horizontally.)

How many foot-pounds of work are required to lift a 600-ton train up this distance against gravity?

- 30. Find the average angle of incline for the roadbed from Altoona, Pa., to Gallitzin, Pa., the rise being about 980 ft. in 10 miles.
- **31.** Two adjacent spokes in a wheel make with one another an angle whose cosine is .91355. What is the angle? How many spokes are there in the wheel?
  - 32. A belt passes over two pulleys whose centers are 11 ft.



apart. The pulley diameters are 18 in. and 42 in. Find the angle between the two sides of the belt.

- 33. Find the angle between the sides of a belt connecting two pulleys 24 in. and 12 in. in diam. and with centers 6 ft. apart.
- 34. Find the length of a side of an equilateral triangle inscribed in a 24-in. diam. circle. Fig. 136 shows the circle and triangle, Drawing the radii OA and OB we have the triangle OAB. Line OC divides this triangle into two equal right triangles OAC and OCB.

Large  $AOC = 60^{\circ}$ . To find side AB we find AC first, then  $AB = \times AC$ . Now in triangle AOC

 $\frac{AC}{OA} = \text{sine } 60^{\circ}$   $AC = OA \times \sin 60^{\circ} = OA \times .866$ 

But OA is the radius of the circle or 12 in.

herefore

'herefore  $AC=12\times.866=10.4$  in. and  $AB=2\times AC=2\times 0.4=20.8$  in. Ans.

35. Find the length of one side of each of the following equalded inscribed figures.

Number of equal sides

Diam of circle in which figure is inscribed

18 in.

7

24 in.

**36.** As shown in Fig. 137, find the distance AB to be laid off for rilling two of eight equally spaced holes on an 8-in. diam. circle.

angle 
$$a = \frac{360^{\circ}}{8} = 45^{\circ}$$
  
angle  $b = \frac{45}{2} = 22\frac{1}{2}^{\circ}$ .

herefore in right triangle BCD

 $\begin{array}{l} \underline{\operatorname{side}BD} \\ \underline{\operatorname{side}BC} \end{array} = \sin \text{ angle } b, \text{ therefore} \\ \\ \underline{\operatorname{side}BD} = \underline{\operatorname{side}BC} \times \sin b. \\ \\ = 4 \text{ in.} \times \sin 22\frac{1}{2}^{\circ}. \\ \\ = 4 \times .383 = 1.532 \text{ in.} \\ \\ \underline{\operatorname{and}} AB = 2 \times BD = 2 \times 1.53 = 3.06 \text{ in.} \quad Ans. \end{array}$ 

37. Find the distance at which to set dividers for laying off 12 qually spaced holes on a 42-in. diam. circle.

38. What is the distance in in. between any two consecutive enters when 17 are spaced an equal distance apart on a 1-ft. am. circle?

39. If the straight distance between two consecutive centers a 12-in. diam. circle is 1½ in., what is the angle formed at the enter of the circle by the radii drawn to these two centers?

40. In the sketch, Fig. 138, showing a cross-sectional view of a

boiler shell 72-in. diam. with water line 20 in. above the center of shell, find the length of the water line "x" in in.

- 41. What is the taper per ft. of length for a wedge, the sides of which form an angle whose tangent is .0626?
  - **42.** In Fig. 139, diameter AB = 2 in., chords AC and AD are

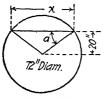


Fig. 138.

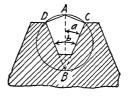


Fig. 139.

each  $\frac{1}{2}$  in. and angles C and D are each 90°. Find angle a and angle b. Angle b is that used in constructing the "acme" thread. Remember how this figure is drawn, to get this angle for an "acme thread."

43. In Fig. 140, find the length of the center lines of rafters A

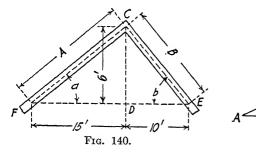


Fig. 141.

and B. (Note.—The tangent of angle  $a = \frac{6}{15} = \frac{2}{5} = .4$ . Find angle a in the tables. Then knowing a and the 15-ft. side of the triangle CDF find side A. In the same way to find side B, find angle b first, then work out the right triangle CDE for side B knowing angle b and the 10-ft. side.

44. A line shaft is 18 ft. above and 15 ft. to the left of a motor

shaft. Find the distance between these shafts by drawing the right triangle, using the tables to get one of the acute angles and then the sine or cosine rule to get the hypothenuse of the triangle or the answer.

45. Find the angle between the belt on two sides of pulleys 18 in. and 42 in. in diam. and centers 11 ft. apart. Make a sketch of the belt and pulleys.

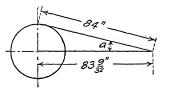


Fig. 142.



Fig. 143.

- 46. Find the length of plank required to reach from A to B, as per Fig. 141.
- 47. In Fig. 142 with crank pin on the top quarter the distance from center of driver to center of cross-head is  $83\%_2$  in. The main rod is 84 in. long. Find the angle a and the stroke of the engine.

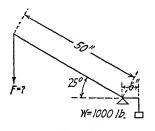
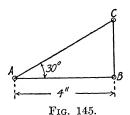


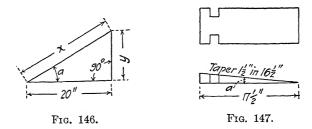
Fig. 144.



48. As in Fig. 143 find to what distance you would set dividers for laying off 26 equally spaced stud holes on a 23½-in. diam. circle on a cylinder head.

**49.** As shown in Fig. 144, what force at *F* is necessary to lift the weight of 1000 lb.?

- **50.** Three holes are to be laid out as in Fig. 145. Find distances BC and AC.
  - **51.** In Fig. 146,  $\cos a = .955$ . Find angle a and sides x and y.
- **52.** The sides of a wrought-iron wedge form an angle of 2° 23′. What is the taper per ft. of length?
- 53. Find the taper per ft. of length of a wedge the sides of which form an angle whose tangent is .0625.



- 54. The shank on a drill socket has a taper of  $\frac{5}{6}$  in. in 12 in., is  $\frac{5}{6}$  in. in diam. at the small end and 5 in. long. What is its diameter at the large end (to the nearest 64th of an in.) and the angle between the sides?
- 55. If a piece of work is to be planed to a taper of  $\frac{1}{16}$  in. in 1 in. for a length of  $\frac{22}{6}$  in., how high would you block up the thin end?

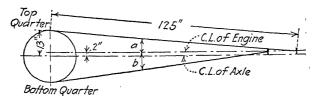


Fig. 148.

- 56. To what distance would you set dividers for laying off 4 equally spaced centers on an 8-in. diam. circle?
- 57. In Fig. 147 showing two views of a wedge find angle a in degrees and minutes. How high would you block up the thin end for planing the tapered surface?

58. By what distance in in. is the center of the cross-head (Fig. 148) drawn back of the middle of its stroke when the crank pin is on both the top and bottom quarters?

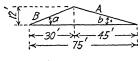


Fig. 149.

**59.** In Fig. 149, find the value of angles a and b and sides A and B.

**60.** For Fig. 150, find the distance between the centers A and B.

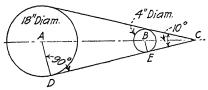
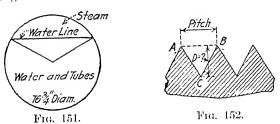


Fig. 150.

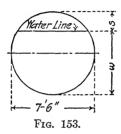
(Note.—Find first the hypothenuse of each of the right triangles ADC and BEC.)

61. Fig. 151 shows a cross-sectional view of a boiler shell 76¾



in. in diam, and the water line is  $21\frac{1}{2}$  in, above the center line of the boiler. Find (1) the area of the steam segment, (2) the area of the water segment, and (3) the net water area when there are 465 2-in, outside diam. (O. D.) tubes.

- 62. Find the depth of a V thread in terms of the number of threads per in. Triangle ABC, Fig. 152, is equiangular and therefore equilateral.
- 63. Fig. 153 represents a cross-sectional view of a boiler shell 7 ft. 6 in. in diam. and has 482 15%-in. O. D. tubes, the ratio of the height of the steam segment to the height of the water



segment or S to W is as 2 is to 7, thus  $\frac{S}{W} = \frac{2}{7}$ . Find (1) the height and area of the steam segment; (2) the height and area of the water segment; (3) the net area of the water segment after deducting area of the tubes.

64. Determine the distance in in. between the center of the

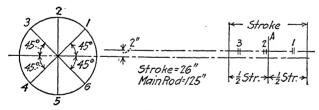


Fig. 154.

cross-head and the center of the stroke or point "A," Fig. 154, when the crank pin occupies the six different positions marked 1-2-3-4-5 and 6. The center line of the cylinder is 2 in. above the center line of the axle. How would these distances be changed, first by shortening the main rod, and second by lengthening the main rod?

## CHAPTER XVIII

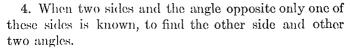
# THE MEASUREMENT OF OBLIQUE TRIANGLES

66. Methods of Working out Oblique Triangles.—In the following problems we are to work out triangles which have no right angles, that is oblique triangles.

How we work out these oblique triangles depends on which sides and angles and how many of either are given, and which sides and angles we wish to find.

In any problem we will have one of the four following cases.

- 1. When one side and two angles are known, to find the remaining two sides and angle.
- 2. When two sides and their included angle are known, to find the other side and other two angles.
- 3. When only the three sides are known, to find all three angles.



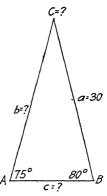


Fig. 155.

## CASE I

Example.—Given, any one side and any two angles of the triangle. To find the other two sides and angle.

In Fig. 155 suppose we have given angle  $\Lambda = 75^{\circ}$ , 177

12

angle  $B=80^{\circ}$  and side a=30. To find angle C and sides b and c we proceed as follows:

In every triangle the three angles added together equal 180°. Therefore angle A + angle B + angle C = 180°. But angle A = 75° and angle B = 80°. Therefore 75° + 80° + angle C = 180° and angle C = 180° - 75° - 80° or 25°.

To find the sides b and c we have the rule. In any triangle any two sides have the same ratio as the *sines* of their opposite angles. Therefore the figure shows

$$\frac{\text{side } b}{\text{side } a} = \frac{\text{sine angle } B}{\text{sine angle } A}$$

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine angle } C}{\text{sine angle } A}$$

also

using the last rule we have

$$\frac{\text{side } c}{30} = \frac{\text{sine } 25^{\circ}}{\text{sine } 75^{\circ}} \text{ or } \frac{\text{side } c}{30} = \frac{.423}{.966}$$

from which side  $c = \frac{30 \times .423}{.966}$  or 13.14 in.

also from the rule that

$$\frac{\text{side } b}{\text{side } a} = \frac{\text{sine angle } B}{\text{sine angle } A}$$

$$\frac{\text{side } b}{30} = \frac{\text{sine } 80^{\circ}}{\text{sine } 75^{\circ}}$$

$$\frac{\text{side } b}{30} = \frac{.985}{.966}$$

From which

and side  $b = \frac{30 \times .985}{.966}$  or 30.6 in.

Example.—In Fig. 156 angle  $A = 56^{\circ}$  10' angle  $B = 64^{\circ}$  30' and side b = 21. We want to find angle C and sides a and c.

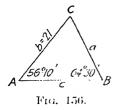
Angle 
$$C = 180^{\circ} - (\text{angle } A + \text{angle } B)$$
  
 $= 180^{\circ} - (56^{\circ} 10' + 64^{\circ} 30')$   
 $= 59^{\circ} 20'$   
side  $a = \sin A$   
side  $b = \sin B$   
side  $a = .830$   
 $21 = .903$   
side  $a = \frac{21 \times .830}{.903}$  or 19.3

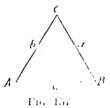
Therefore

To find side c, we have the rule:

$$\frac{\text{side } c}{\text{side } b} = \frac{\sin C}{\sin B} \text{ therefore } \frac{\text{side } c}{21} = .860$$

from which, side  $c = \frac{21 \times .860}{.903}$  or 20.0





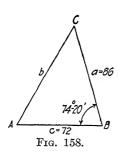
1. From the explanation just given and u mg log log, complete the following table, drawing a separate triangle for each problet. The answers are given to the first two problems to help mander standing them.

	Angle .1	Angle  B	Viii-le (		+ 4 +	
1	60"	65 1207	[54 ° 40°	18 m	1   1 = 0 121 1	11, 11
$\frac{2}{3}$	80"	30 ' 30'	(69 30)	, 16 G m	24 10	11 1
3	70"		10 .00			,1,
4		18	72		1 .7 11.	
4 5	37"	87 451		18.35		
6		$42 - 16^{\circ}$	5.4			
7		30	ti()			

## CASE II

Example.—Given, any two sides and their included angle. To find the remaining side and remaining angles.

In Fig. 158 we have given side a = 86, side c = 72,



and angle 
$$B = 74^{\circ} 20'$$
. In order to find side  $b$  and angles  $A$  and  $C$  we use the following rules.

$$\frac{\tan \frac{1}{2}(\text{angle } A - \text{angle } C)}{\tan \frac{1}{2}(\text{angle } A + \text{angle } C)} = \frac{\text{side } a - \text{side } c}{\text{side } a + \text{side } c}$$

Now angle A + angle B + angle  $C = 180^{\circ}$ 

Therefore Angle  $A + \text{Angle } C = 180^{\circ} - \text{Angle } B$  or  $180^{\circ} - 74^{\circ} 20'$  or  $105^{\circ} 40'$ .

Then using the rule above, we have

$$\frac{\tan \frac{1}{2} (\text{angle } A - \text{angle } C)}{\tan \frac{1}{2} (105^{\circ}40')} = \frac{86 - 72}{86 + 72}$$

or tan ½(angle A – angle C) = tan ½(105° 40′)  $\times \frac{14}{158}$ .

The tan  $\frac{1}{2}(105^{\circ} 40') \times \frac{14}{158} = \tan 52^{\circ} 50' \times \frac{14}{158}$  or  $\frac{1.319 \times 14}{158}$ . This fraction worked out equals .117.

If  $\tan \frac{1}{2}(\text{angle }A - \text{angle }C) = .117$  we find from the table of tangents that  $\frac{1}{2}(\text{angle }A - \text{angle }C) = 6^{\circ}$  40' and angle  $A - \text{angle }C = 2 \times [6^{\circ}$  40'] or 13° 20', but angle  $A + \text{angle }C = 105^{\circ}$  40'. Adding, 2 × angle  $A = 119^{\circ}$  and angle  $A = 59^{\circ}$  30'.

Angle 
$$C = 180^{\circ}$$
 — angle  $A$  — angle  $B$  or  $180^{\circ}$  —  $59^{\circ}$   $30'$  —  $74^{\circ}$   $20'$  or  $46^{\circ}$   $10'$  To find side  $b$  we have as in Case I.

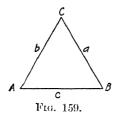
$$\frac{\text{side } b}{\text{side } a} = \frac{\sin B}{\sin A}$$

That is

$$\frac{\text{side } b}{86} = \frac{\sin 74^{\circ} 20'}{\sin 59^{\circ} 30'}$$
  
side  $b = \frac{86 \times .963}{.862} = 96.1$ 

Therefore

2. From the explanation just given and using Fig. 159, complete the following table, drawing a separate triangle for each problem.



The answers in parenthesis() are given for the first two problem to help in understanding them.

	Angle A	${\rm Angle}\ B$	Angle C	Side a	Side b	Side $c$
1	(116° 10′)	40° 20′	(23° 30′)	72	(51.9)	32
2	58° 40′	$(70^{\circ})$	(51° 20′)	(5.90)	65	5.4
3			70°	120	110	
4	36° 40′				98	103
5		86°		18		24
6			98° 20′	7.92	12.3	
7	88° 40′				2.35	4.2
8		23"		56		63

## CASE III





(1) 
$$\cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$
 (2)  $\cos \frac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}}$  (3)  $\cos \frac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}$ 

In Fig. 160 we have  $s = \frac{\text{side } a + \text{side } b + \text{side } c}{2} = \frac{4+6+8}{2}$  or 9, and using rule (1) above:

 $\cos \frac{1}{2}A = \sqrt{\frac{9(9-4)}{6\times8}} \text{ which equals } \sqrt{\frac{9\times5}{48}} \text{ or .9683.}$ 

If  $\cos \frac{1}{2}A = .9683$  from the table of cosines we find  $\frac{1}{2}A = 14^{\circ} 28'$ , and therefore  $A = 28^{\circ} 56'$ .

By rule (2) above,  $\cos \frac{1}{2}B = \sqrt{\frac{9(9-6)}{4\times8}}$  which equals  $\sqrt{\frac{27}{32}}$  or .9185, and from the tables  $\frac{1}{2}B = 23^{\circ}$  17′. Therefore  $B = 46^{\circ}$  34′. Then angle  $C = 180^{\circ}$  – (angle  $A + 180^{\circ}$ )

angle B) or  $180^{\circ} - (28^{\circ} 56' + 46^{\circ} 34')$  which equals 104° 30′.

3. Using Fig. 161 and from the explanation just given complete the following table, drawing a separate triangle for each problem. The answers to the first problem are already given.

	Side a	Side b	Side c	Angle A	Angle B	Angle C
1 2 3 4 5 6 7 8	3 4 12 5.6 5.8 60 70 56	5 6 10 • 4.8 6.5 80 62 43	6 5 9 3.7 8.4 48 39 36	(29° 56′)	(56° 16′)	(93° 48′)

Put on each triangle the lengths of the sides given and the values of the angles found.

## CASE IV

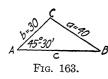
Example.—Given any two sides of the triangle and the angle opposite one of them To find the remaining side and other two angles. Suppose, as in Fig. 162, we have given side a = 24, side b = 30.5and angle  $A = 31^{\circ} 20'$ . To find angle B we have by Case I:

Fig. 162.

$$\frac{\sin B}{\sin A} = \frac{\text{side } b}{\text{side } a}$$
 or  $\sin B = \frac{30.5 \times .520}{24}$  or .6608

and from the table of sines we find angle  $B = 41^{\circ} 22'$ . But an angle and its *supplement* have the same sine, the supplement of an angle being that angle which added to the given angle will equal 180 degrees. For example, suppose we have given an angle of  $38^{\circ} 40'$ , then  $180^{\circ} - 38^{\circ} 40' = 141^{\circ} 20'$  and  $141^{\circ} 20'$  is the supplement of the angle  $38^{\circ} 40'$ .

It is possible, therefore, to have two values for angle B in this problem. Hence another value for angle B in this problem is  $180^{\circ} - 41^{\circ} 22'$  or  $138^{\circ} 38'$ . Call this value  $B_1$ ; and since we have two values for angle B, we must have two triangles, each having an angle  $A = 31^{\circ} 20'$ , a side a = 24, and a side b = 30.5, and hence two



values for angle C and side c. Call these values C and  $C_1$ . Then  $C = 180^{\circ} - (A + B)$  or  $180^{\circ} - (31^{\circ} \ 20' + 41^{\circ} \ 22')$  which equals  $107^{\circ} \ 18'$ ; also  $C_1 = 180^{\circ} - (A + B_1)$  or  $180^{\circ} - (31^{\circ} \ 20' + 138^{\circ} \ 38')$  or  $10^{\circ} \ 2'$ .

We have now to find side c and side  $c_1$ . By Case I we have

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A} \text{ and } \frac{\text{side } c_1}{\text{side } a} = \frac{\text{sine } C_1}{\text{sine } A}$$
Therefore side  $c = \frac{24 \times .95476}{.520} \text{ or } 44.06$ 
and side  $c_1 = \frac{24 \times .17422}{.520} \text{ or } 8.04$ 

We will not always have two solutions, that is, two values for the unknown angles and sides. For instance, suppose as in Fig. 163 we had given side a = 40, side b = 30 and angle  $A = 45^{\circ} 30'$ . Then by Case I we have

$$\frac{\text{sine } B}{\text{sine } A} = \frac{\text{side } b}{\text{side } a}$$
 or sine  $B = \frac{30 \times .71325}{40}$  or .5349

and from the table of sines we have  $B = 32^{\circ} 20'$ .

The supplement of angle B is  $180^{\circ} - 32^{\circ} 20'$  or  $147^{\circ} 40'$ , that is, angle  $B_1$ . But the angles  $A + B_1 = 45^{\circ} 30'$ 

+ 147° 40′ or 193° 10′ which is impossible, since the sum of any two angles of a triangle must be less than 180°, hence we have only one solution to the problem: Angle  $C = 180^{\circ} - (A + B)$  or  $180^{\circ} - (45^{\circ} \ 30' + 32^{\circ})$ 20') or 102° 10', since

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A}$$
, side  $c = \frac{40 \times .97754}{.71325}$  or 54.8

Again, suppose as shown in Fig. 164 we had given side a = 32, side b = 44, and angle  $B = 112^{\circ} 30'$ . Then by Case I we have  $\frac{\sin a}{\sin B} = \frac{\text{side } a}{\text{side } b}$ 

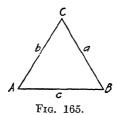
Now sine  $112^{\circ} 30' = \text{sine } (180^{\circ} - 180)^{\circ}$ 112° 30′) or sine 67° 30′ (since the sine of any angle less than 180°. equals the sine of 180° minus that an-Therefore sine  $A = \frac{32 \times .92388}{44}$ or .67191, and from the table A =42° 13′. Angle  $C = 180^{\circ} - (A + B) A$ or  $180^{\circ} - (42^{\circ} 13' + 112^{\circ} 30')$  or  $25^{\circ}$ Fig. 164. 17'; also since  $\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A}$ , then side  $c = \frac{32 \times .42709}{.67191}$  or 20.34.

It is evident, since the given angle is greater than 90°, that there is only one solution to this problem.

We now have the following rules by which we may tell at a glance how many solutions there are to a problem under this case (Case IV).

- 1. If the side opposite the given angle is less than the other given side there are two solutions.
- 2. If the side opposite the given angle is greater than the other given side there is only one solution.

- 3. If the given angle is greater than 90° there is only one solution.
- 4. From the explanation just given and using Fig. 165 as a guide complete the following table, drawing a separate triangle for each problem. The answers to the first problem are already worked out.

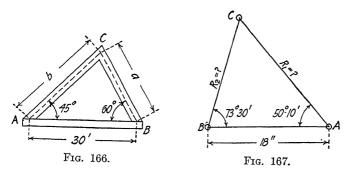


	Angle A	Angle B	Angle C	Side a	Side b	Side c
1	32° 10′	$B = 45^{\circ}13'$ $B_1 = 134^{\circ}47'$	C = 102°37' $C_1 = 13°3'$	. 6	8	$c = 11$ $c_1 = 2.54$
2		11	26° 40′		10	7
3		102° 50′			73	6 <b>5</b>
4			39° 10′	28		33
5		59°		16	14	
6		120° 50′			48	42
7			83° 20′	47		51
8	128° 30′			38		30

- 67. Applications of Triangular Functions to Oblique Triangles.—Following are applications to the solution of practical problems of the preceding rules for oblique triangles.
- **5.** In Fig. 166 is shown the center lines of a roof truss. Find the angle C and the length of the rafters a and b.
- **6.** In Fig. 167 find the radii  $R_1$  and  $R_2$  for laying out the three centers A, B, and C so that they may be located as shown.

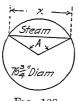
7. In Fig. 168 the diameter of a boiler is given as 76¾ in. and angle "A" is 150°. Find the length of water line "x."

8. It is desired to find the distance between two points A and B on opposite sides of a river. If the distance from B to a point C, Fig. 169, is found to be 128 ft. and the angles ABC and BCA are



found to be 43° 40′ and 50° 20′ respectively, what is the distance AB?

9. It is desired to find the height of the stack shown in Fig. 170. The angle of elevation of a point in the horizontal plane





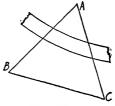


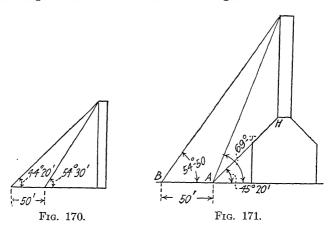
Fig. 169.

of the base of the stack is 54° 30′, and the angle of elevation at a point 50 ft. farther away is 44° 20′. Find the height of the stack.

10. The angle of elevation of the top of a stack at a point A, as shown in Fig. 171, is 69° 20′ and the angle of elevation at a point B, 50 ft. farther away, is 54° 50′. If the angle of elevation at

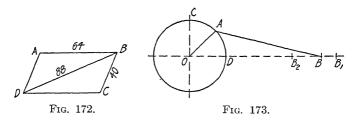
point A of the top of the building is  $45^{\circ}$  20', find height of stack and height of building.

11. Two trains start from the same station at the same time traveling on tracks which intersect at an angle of 64° 20′. If the



trains travel at the rate of 35 and 50 m.p.h., respectively, how far apart are they at the end of 45 min.?

12. A 1/4-in. steel plate has the form of a parallelogram as shown in Fig. 172. If the sides are 64 in. and 40 in. in length and the

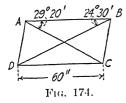


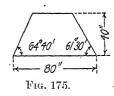
longer diagonal is 88 in. in length, find the angles A, B, C, and D, the area of the plate, its volume, and also its weight if 1 cu. in. of the metal weighs .283 lb.

13. In a steam engine the crank OA (Fig. 173) is 13 in. long, and

the connecting rod AB is 125 in. long. How far has the center B of the cross-head moved from its head end dead center position  $B_1$  when the crank pin is midway between C and D?

14. Given a ½-in, steel plate with the shape as shown in Fig. 174. Find its area in sq. in, and its weight if steel weighs .28 lb, per cu. in.





15. Given a 3g-in, steel plate as shown in Fig. 175 with its longest side 80 in, and width 40 in. Find the non-parallel sides, its area in sq. in., and its weight.

### CHAPTER XIX

### ELECTRICITY

68. Definitions and Calculation of Resistance.—In making electrical calculations we need to understand the meaning of electrical "resistance" and how to calculate Every wire or electric circuit or portion of a circuit which conducts an electric current offers a "resistance" to the flow of the current through it. We may consider this resistance in a way like mechanical friction. Less water will flow in a pipe that is rough than in a smooth pipe because friction cuts down the flow. Less electric current will flow in a wire or "conductor," as it is called, having a high resistance than in a low-resistance conductor. The effect of the electrical resistance is to cause a part of the electrical energy to be changed into heat. Every electrical conductor is therefore heated by an electric current flowing through it. The less the resistance of a conductor the less the heat produced by a given current. Electrical resistance is measured in ohms.

The resistance of a conductor depends on its material, that is, whether copper, aluminum or iron, upon its length, area of cross section and its temperature. Copper is a better conductor of electricity than either aluminum or iron, that is, it offers less resistance to the flow of the electric current through it. If copper wire is heated its resistance increases. The resistance of a wire increases as its length is increased and decreases as its area of

cross section is increased. In electric wiring it is necessary to have wire large enough in cross section to carry the current required without overheating. If a wire too small is used there is danger from fire and insurance rules limit the size of wire for different kinds of work. In calculating the resistance of wires of circular cross section, their area is often found in circular mils, by squaring the diameter expressed in thousandths of an inch. Thus a wire 14 in. or .25 in. in diam. has  $(.25 \times 1000)^2$  or (62,500) circular mils in it. This is not the area as we determine the area of a circle, but is a convenient way of expressing relative areas of wire cables, etc. To find the resistance of copper wire at ordinary temperature we use the following rule:

Resis, in ohms = 10.8 × length of wire in ft. area of cross section in cir. mile

Using this rule the resistance in ohms of 1 mile  $\pm 5280$  ft.  $\pm$  of copper wire .12 in. in diam, is equal to

$$10.8 \times 5280$$
 or  $10.8 \times 5280$   $3.96$  ohm.

For another example, the resistance of 2000 ft, of copper wire .20 in, in diam, is

$$10.8 \times 2000 \text{ or } 10.8 \times 2000 \text{ or .54 ohm}$$
  
 $(.20 \times 1000)^2 \text{ or .40,000}$ 

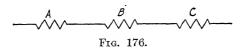
Ten miles of a 100,000 circular mil copper cable has a resistance of

$$10.8 \times 10 \times 5280$$
 or 5.70 ohm

69. Resistances in Series. If a number of electrical conductors are arranged one after another in a current c in Fig 176 so that the same current flow—through all of

them, they are said to be "in series" and their total resistance is equal to the sum of their separate resistances:

Total resistance = resistance of A + resis. of B+ resis. of C



70. Resistances in Parallel.—If the conductors are arranged side by side as in Fig. 177 so that the current divides in going through them and each conductor takes only a part of the total current, they are said to be "in parallel" and the reciprocal of the total resistance is equal to the sum of the reciprocals of the separate resistances:

$$\frac{1}{\text{Total resistance}} = \frac{1}{\text{resis. } A} + \frac{1}{\text{resis. } B} + \frac{1}{\text{resis. } C}$$

$$\frac{A}{C}$$
Fig. 177.

For example, if a copper cable of .10 ohm resistance is in parallel with an aluminum cable of .85 ohm resistance the combined resistance is found as follows:

or 
$$\frac{1}{\text{total resistance}} = \frac{1}{.10} + \frac{1}{.85}$$

$$\frac{1}{\text{total resistance}} = 10 + 1.18 \text{ or } 11.18$$
and total resistance =  $\frac{1}{11.18}$  or .0895 ohm.

or

For transmitting power, copper, aluminum, steel, and phosphor bronze wire are used, copper and aluminum principally. For telephone and telegraph work steel and iron wires are used. For protective devices such as fuses and for coils of electrical measuring instruments, various alloys or mixtures of metals are used.

Copper is used to a large extent as an electrical conductor on account of its comparatively low resistance. Aluminum has an advantage over copper in its lighter weight for the same conducting power. This in transmission lines reduces the weight on insulators, poles, and cross-arms.

### PROBLEMS ON RESISTANCE

- 1. Find the resistance of 5 miles of transmission line with conductors .187 in. in diam.
- 2. It is desired to extend the above line 2 miles with conductors .125 in, in diam. What is the resistance of the line added and of the total line?
- 3. Find the resistance of 12 miles of 150,000 circular mil copper cable.
- **4.** An aluminum cable of 12 ohms resistance is put "in parallel" with the copper cable of Prob. 3. What is the resistance of the two in parallel?
- **5.** A copper wire has a resistance of 20 ohms. Another wire of the same material is twice as long as the first and twice as large in cross section. How does its resistance compare with that of the first?
- 6. Give as many examples as you can of the use made of the heating effect of an electric current flowing through a resistance.
- 7. The resistance of a line wire is 4 ohms and a second wire of the same size is run along and "tied in" with the first, what is the resistance of the two taken together?
- 8. The resistance of a copper wire is 5 ohms. What is the resistance of another wire of the same material three times as long and of one-third the cross-sectional area of the first?

- 9. What metals are used as electrical conductors besides copper and aluminum? Explain in what kind of service they are used.
- 10. From the table on this page what number wire would you use for concealed work to carry a current of 22 amperes? What part of an inch is the diameter of this wire, and what does it weigh per mile if insulated?

71. Safe Carrying Capacity of Wires.—

Gauge No., B. & S.	Diameter, mils.	Ohms per 1000 ft.	No. amperes, open work	No. amperes, concealed work	Lb. per 1000 ft., bare	Lb. per 1000 ft., insulated
18	40	6.3880	5	3	4.92	18
17	45	5.0660	6	4	6.20	21
16	51	4.0176	8	6	7.82	25
15	57	3.1860	10	8	9.86	31
14	64	2.5266	16	12	12.44	38
13	72	2.0037	19	14	15.68	43
12	81	1.5890	23	17	19.77	48
				(		
11	91	1.2602	27	21	24.93	<b>64</b>
10	102	.99948	32	25	31.44	80
9	114	.79242	39	29	39.65	97
8	128	.62849	46	33	49.99	116
7	144	.49845	56	39	63.03	118
6	162	.39528	65	45	79.49	166
5	182	.31346	77	53	100.23	196
4	204	. 24858	92	63	126.40	228
3	229	.19714	110	75	159.38	265
$^{2}$	258	.15633	131	88	200.98	296
1	289	.12398	156	105	253.43	329
0	325	.09827	185	125	319.74	421
00	365	.07797	220	150	402.97	528
000	410	.06134	262	181	508.12	643
0000	460	.04904	312	218	640.73	815
	1					

72. Ohm's Law, Calculation of Current, Voltage and Resistance.—In order to force an electric current through

a wire or conductor in spite of its resistance it is necessary to have an electric pressure just as it is necessary to have water pressure or "head" to force water through a pipe in spite of friction.

Electric pressure is measured in *volts* and is indicated by an instrument called a *voltmeter*. Instead of electric

pressure we sometimes speak of the "voltage" of a circuit or the "electromotive force" of the circuit. The three terms "electric pressure," "voltage," and "electromotive force" all mean the same pressure required to force an electric current through a circuit. The amount of current which will flow in a circuit of given resistance and under a given pressure is found

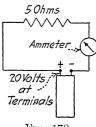


Fig. 178.

from the following rule known as Ohm's Law (named from Dr. Ohm):

$$Current = \frac{\text{voltage}}{\text{resistance}}$$

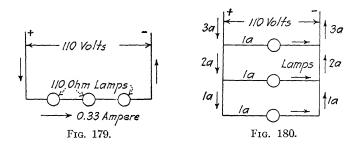
If the voltage is in "volts" and the resistance in "ohms" the current is in "amperes." For example, if, as in Fig. 178, a 20-volt battery is connected to a wire of 5 ohms resistance a current of  $\frac{20}{5}$  or 4 amperes will flow through the circuit, and may be read on the "current meter" or "ammeter."

For example, if an incandescent carbon filament lamp having a resistance of 220 ohms is put on a 110-volt circuit, the current which it will take equals  $\frac{110}{220}$  or .5 ampere. From the rule that

- (1) Current =  $\frac{\text{voltage}}{\text{resistance}}$  it follows that
- (2) Voltage = current × resistance, and also
- (3) Resistance =  $\frac{\text{voltage}}{\text{current}}$ .

Rule (2) enables us to find the voltage across a known resistance through which a known current flows, and rule (3) enables us to find the resistance of a circuit with a known current flowing under a known pressure.

In Fig. 179, three lamps are arranged "in series" across 110-volt mains. Here the current through each



lamp is the same and if each lamp has a resistance of 110 ohms, the three "in series" have a resistance of  $3 \times 110$  or 330 ohms. The current through the lamps therefore equals  $\frac{\text{the voltage}}{\text{the resistance}}$ , or  $\frac{110}{330}$  or .33 ampere.

From rule (2) the voltage across each lamp equals current  $\times$  resistance, or .33  $\times$  110 or 36.7 volts.

Fig. 180 shows three 110-ohm lamps arranged "in parallel" across 110-volt mains. The current through each lamp equals  $\frac{\text{the voltage}}{\text{resistance of lamp}} = \frac{110}{110}$  or 1 ampere. The three lamps together therefore take three amperes.

The arrows indicate the direction of flow of the current and the figures, the number of amperes in the different parts of the circuit.

### **PROBLEMS**

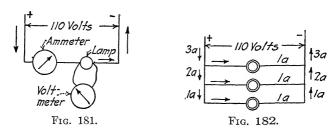
- 11. What is the resistance of a magnet winding through which 1.5 amperes of current flows under a pressure of 110 volts? Draw a figure showing the magnet in circuit?
- 12. If a 32-e.p. carbon lamp of 110 ohms resistance is connected across a 110-volt circuit, what current flows through it? How much current do ten of these lamps take when arranged "in parallel" on a 110-volt circuit?
- 13. How much current would each of ten 220-ohm lamps take if put "in series" in a 550-volt circuit?
- 14. A carbon filament lamp operating on 110 volts takes .95 ampere when first put in circuit and .93 ampere after burning for 10 min. What is the change in resistance due to heating the filament?
- **15.** Three 110-ohm lamps are connected in series and these in series with a *parallel* group of five 220-ohm resistances. What is the resistance of the total number of lamps as arranged?
- 16. In Prob. 15 what is the voltage across each part of the circuit when 110 volts are impressed on the total circuit described?
- 17. Two thousand feet of No. 18 B. & S. gauge copper wire is to be wound on a magnet and operated on a 110-volt circuit. What resistance must be put "in series" with this winding in order that the current may be kept within safe limits? (See table, page 194.)
- 18. What is the weight of 40 miles of No. 00 B. & S. gauge trolley wire?
- 19. What is the total current taken by ten lamps in parallel if each has a resistance of 121 ohms and operates on a 110-volt circuit?
- 73. Calculation of Power and Energy in an Electric Circuit. The power consumed in a direct current circuit and the power taken by an incandescent lamp, whether on a direct or alternating current circuit equals the product

of the current flowing through the circuit and the voltage across the circuit, that is,

$$power = current \times voltage$$

When the current is in "amperes" and the voltage in "volts," the *power* is in watts.

For example, if an incandescent lamp, as in Fig. 181, across 110-volt mains and has 1 ampere flowing



through it, the *power* in watts equals the voltage 110 multiplied by the current 1 or watts =  $110 \times 1$  or 110.

In Fig. 182, three 110-ohm lamps are shown in parallel across 110-volt mains. By the rule for finding the current, we have the current taken by each lamp =

$$\frac{\text{the voltage across lamp}}{\text{resistance of the lamp}}$$
 or  $\frac{110}{110} = 1$  ampere.

The power taken by each lamp = current through lamp  $\times$  voltage across lamp =  $1 \times 110$  or 110 watts. The three lamps together therefore take  $110 \times 3$  or 330 watts. The arrows and numbers in the figure show the direction of flow of the current and the number of amperes in each part of the circuit.

One thousand watts is called a kilowatt. In the

example above 330 watts or  $\frac{330}{1000} = .330$  of a kw. is the power consumed constantly.

The energy consumed in an electric circuit equals the product of the power and the time during which the power is taken. If the power is in watts and the time in hours, their product gives the energy in watt-hours. If the power is in kilowatts and the time in hours their product gives the energy in kilowatt-hours which is the unit used by electric light and power companies. If lights which take .30 kw. are kept burning for 3 hr. the energy they consume in this time equals kilowatts  $\times$  hours or .30  $\times$  3 or .90 of a kw.-hr.

Electrical energy is sold at prices ranging from a fraction of a cent to 18 or 20 cts. per kw.-hr. (kilowatthour). At 8 cts. per kw.-hr. the cost of the energy consumed by the lamps taking .90 kw.-hr. is .90  $\times$  8 or 7.2 cts.

### **PROBLEMS**

- 20. What power is taken by a 110-volt carbon filament lamp with a current of .98 ampere?
- ${\bf 21.}\,$  A tungsten lamp takes .545 ampere at 110 volts. What is its wattage?
- 22. What is the difference in power taken by a group of five 110-volt carbon filament lamps each taking 1 ampere and a cluster of three 110-volt tungsten lamps each taking 60 watts?
- 23. Calculate the kilowatt-hours of energy consumed in 5 hr. by three 100-watt tungsten lamps.
- **24.** How much energy is taken in 2 hr. by twelve 60-watt lamps?
- 25. What is the cost per month of 30 days of 2 hr. of lighting per day, for an entire building with energy of 8 cts. per kw.-hr., if there are twelve 100-watt lamps and the equivalent of twenty 40-watt lamps in the building?
  - 26. Find the cost per month (30 days) for house lighting, with

4 carbon filament lamps each taking 1 ampere at 110 volts and burning for an average of 5 hr. per day. Take the cost of energy at 8 cts. per kw.-hr.

- 27. What kinds of incandescent lamps are in general use? State the advantages and disadvantages of each.
- 28. An office is first lighted with six 16-c.p. carbon filament incandescent lamps taking 3.1 watts per candle. The carbon lamps are later replaced by four 60-watt tungsten lamps. What is the saving per month of 30 days with the lamps burning 5 hr. per day if energy costs 8 cts. per kw.-hr.?
- 29. How does the first cost of carbon and tungsten lamps compare? Is there a saving in using tungsten rather than carbon lamps?
- 74. Electrical Horsepower.—Horsepower means the rate at which work is done. For example, 33,000 ft.-lb. of work done per min. or 550 ft.-lb. per sec. represents one horsepower. In electrical terms one horsepower (abbreviated h.p. or HP) equals 746 watts. To find the horsepower taken by a direct-current motor multiply the voltage on the motor by the current taken by the motor and divide the product by 746, that is

$$horsepower of a motor = \frac{Volts on motor \times current intake}{746}$$

Example.—Find the horsepower consumed by a 110-volt motor taking a total current of 10.3 amperes.

Horsepower = 
$$\frac{110 \times 10.3}{746}$$
 or 1.52 Ans.

To find the horsepower delivered by an electric generator, multiply the voltage at the generator terminals by the current output and divide the product by 746, that is

horsepower of a generator 
$$=\frac{\text{Volts at gen. ter.} \times \text{curr. out.}}{746}$$

Example.—An electric generator with 110 volts at its terminals supplies 25 amperes to a lamp board. What horsepower is it giving out as useful power?

Horsepower = 
$$\frac{110 \times 25}{746}$$
 or 3.69 Ans.

To find the *horsepower-hours*, multiply the load in horsepower by the time in hours during which the power is supplied.

Example.—A generator supplies 13 amperes at 120 volts for 5 hr. what is its horsepower hour output?

Horsepower hours = 
$$\frac{120 \times 13}{746} \times 5 = 10.45$$
 Ans.

#### PROBLEMS

- **30.** What horsepower generator is necessary to supply 30 amperes at a pressure of 110 volts?
- 31. A motor driving a lathe takes 6.4 amperes at 220 volts, what is the horsepower supplied to the motor?
- 32. Three direct-current and voltage ratings are as follows: 10 amperes, 110 volts; 15 amperes, 220 volts; 30.5 amperes at 110 volts. Find the corresponding horsepower rating of each of these motors when working at full load.
- 33. Five tungsten lamps, each take .95 ampere at 110 volts, what is the number of watts and horsepower taken by the lamps? If the lamps burn for 5 hr. what energy do they take in watt-hours, kilowatt-hours, and in horsepower-hours?
- 34. What kind of instruments are used for measuring an electric current? Find out over what range these instruments are made to read.
- **35.** What kind of instruments are used for measuring electric pressure or voltage? Find out what range these instruments have.
  - **36.** How are electrical "power" and "energy" measured?
- 37. Find out what different kinds of electric lamps are in general use, with the advantages and disadvantages of each.
- 38. What rules do you know in regard to personal safety and protection of machines when working about electrical circuits?

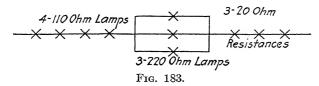
- 75. Methods of Charging for Electrical Energy.—
- (a) Flat Rate System.—In the flat rate system the customer is charged a certain amount per month for each lamp installed. No meters are used to determine the amount of energy consumed. With this system a wasteful consumer pays no more for the same number of lights installed than an economical user. This system saves the cost of meters, their repair, and simplifies the company's bookkeeping. A scale of discounts is usually arranged increasing with the number of lamps installed.
- (b) METER SYSTEM.—In this system the consumer is charged for the energy which he uses, that is, for the number of kilowatt-hours which he consumes in a certain time. This method would seem to be a just way of charging for energy, but the time of day that power is taken is of as much importance as the amount taken. Power taken at a time when the station load is heavy is more expensive for the company than power taken at a time of light load. The cost of power depends on the size of the power station, the amount of power taken, and the time at which it is taken. In view of these facts a "two rate" meter is sometimes used with two sets of dials, one reading at times of light station load and the other at times of heavy station load. The consumer is then charged one rate for the power taken when the station is working light and a higher rate for power taken at times of heavy load on the station.

A "maximum rate" meter is also used in some cases. This meter gives the greatest load or power which the consumer has taken during a certain period. The charge is then made on a basis of the greatest amount of power taken at any one time as well as on the total number of kilowatt-hours consumed.

A certain charge is usually made whether the customer uses any power or not. This is because the customer has power at hand any time he wishes to use it, and a charge is made for this service.

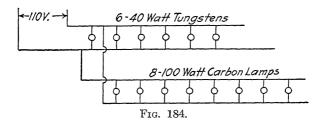
### **PROBLEMS**

- 39. Find the total resistance of the circuit shown in Fig. 183.
- 40. If the circuit of Prob. 39 is connected across 110-volt mains, what is the current through the circuit and the voltage across each portion of the circuit?



- 41. Find the difference in power taken by eight 220 ohm lamps operating on a 220-volt circuit and sixteen 100-watt lamps.
- 42. If a current of 8.2 amperes flows through a resistance on 110 volts, how much more resistance must be put in circuit to reduce the current to 4.5 amperes?
- **43.** How many horsepower-hours are equivalent to 2000 kw.-hr.? How many kilowatt-hours are consumed by a lamp load of 125 amperes at 220 volts in circuit for 4 hr.?
- **44.** Calculate the power in watts taken by thirty 110-volt carbon lamps connected in parallel if each lamp takes 1.5 amperes.
- 45. Calculate the power taken by 20 lamps in series on a 1100-volt circuit if 15 amperes flow through the circuit.
- 46. Calculate the power in kilowatts taken by ten 110-volt lamps in parallel across a 110-volt circuit if each takes .9 ampere.
- **47.** Ten lamps taking 1 ampere each are connected in parallel across 110-volt mains. What is the resistance of each lamp and the power taken by the circuit?
- **48.** Four 110-volt carbon lamps taking 1 ampere each were replaced by four 40-watt tungsten lamps. If the cost of energy was 8 cts. per kw.-hr., how much did they gain or lose per hour by the change?

- 49. Find the difference in kilowatt-hours taken in 5 hr. by ten 220-ohm lamps on 110-volts and eight 60-watt tungstens.
- 50. Find cost of operating the circuit shown in Fig. 184 per month of 30 days and 4 hr. per day, energy 10 cts. per kw.-hr.
- 51. What system of charging for electricity do you consider the best, and why?



76. Comparison of Water and Electric Systems.—A very close and interesting comparison can be made between a water system with its piping, gauges, pumps, motors and so forth and an electric system with its wiring, meters, generators, motors and so forth. In Figs. 186 and 187 diagrams are shown of typical water and electric systems, with the corresponding parts in the two systems marked and explained as follows:

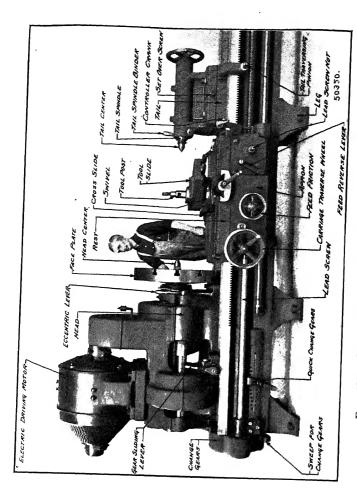


Fig. 185.—Cut showing a lathe with an electric motor drive.

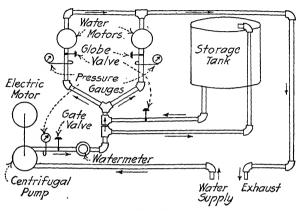


Fig. 186.

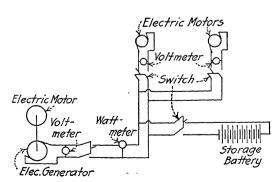


Fig. 187.

Water system	Corresponding part in electric system		
Electric motor for driving centrifugal pump.	Electric motor for driving electric generator.		
Centrifugal pump.	Electric generator.		
Pressure gauge, giving water pressure.	Voltmeter giving electric pressure.		
Gate valve for shutting off water.	Switch for opening electric circuit.		
Water meter for measuring quantity of flow of water.	Wattmeter for measuring the power taken by the circuit.		
Globe valve for shutting off water supply to motor.	Switch for cutting off electric current to electric motor.		
Water motor for doing mechanical work.	Electric motor for doing me- chanical work.		
Storage tank for water pumped by centrifugal pump.	Storage battery for electricity supplied by generator.		

### CHAPTER XX

## STRENGTH OF MATERIALS

77. Definition.—All materials have a property or characteristic which we call "strength," which is quite different for different materials such for example as wood and steel. In order to know what kind of material to use for a particular piece of work and how much of it to use, that is how large a piece is required in order that the completed work or job shall be sufficiently strong to serve its purpose, it is necessary to know what the strength of the various materials is and how this strength differs for the different methods in which pressure may be brought on the material.

The average person is likely to speak only relatively regarding the strength of materials, namely that one material is so many times stronger than another. It is necessary, however, in working with machines and in general construction work to have more definite knowledge of the strength of materials. This has led to the construction of testing machines, capable of measuring directly in pounds the strength of the materials under various conditions. These machines are described later on in this chapter.

The object of this chapter is to convey some idea of the strength of materials, from results secured by actual tests and how these results are used in calculating the size and amount of material required. This chapter also brings out some of the effects of forces applied to materials in changing their size and shape, and also in merely producing pressure in them without changing their form.

78. Stress and Strain.—Whenever a pressure or a load is put upon any piece of material it tends to change the shape of the piece. The material itself resists this change of shape and in so doing exerts a force opposite to the load or pressure applied to the material. For example, if we take a piece of string and, holding it in our hands, pull on it with an increasing force the string will finally break, but not until it has straightened out and become taut. After this there is a gradual stretch until the fibers pull apart. Our effort or pull on the string has been opposed by a resistance in the fibers of the string. This resistance is called stress. Stress may therefore be defined as the resistance which a material offers to the action of an external force, pressure or load which tends to change the shape of the material.

If a 2-ton weight is being lifted by a rope the stress produced in the rope is 4000 lb.

The stretching of the string lengthened or *elongated* it and produced a change of its form. This change of form in the fibers of a material is called *strain*.

In construction work the materials used must be of sufficient size and strength so that the loads applied are not sufficient to change their shape, otherwise the parts are likely to break or "rupture" and disastrous results follow.

- 79. Kinds of Stresses.—There are three general kinds of stresses, namely (1) Tensile stress (or pulling stress),
- (2) Compressive stress (pushing or crushing stress),
- (3) Shearing stress (cutting stress).

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There are two other kinds of stresses which in reality are combinations of those given above. These are (a) Torsion or twisting which is a kind of shearing stress

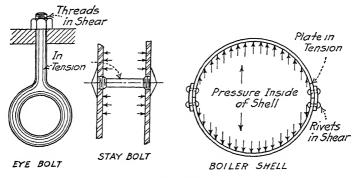
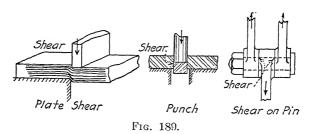


Fig. 188.

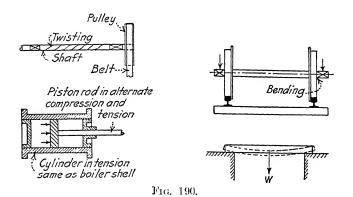
and (b) Bending or flexure which is a combination of tension and compression with or without shearing. The examples (Figs. 188, 189 and 190) show the various stresses of materials in service.



Since the tensile stress set up in a material is due to a pull, the *tensile strength* of the material is a measure of the resistance offered by its fibers to being pulled apart.

Since the compressive stress is a stress set up in a material due to a push, thrust, or crushing force, the *compressive strength* is a measure of the resistance offered by the fibers of the material to being crushed.

A shearing stress is one set up in a material due to a force acting at right angles to the material and the shearing strength is a measure of the resistance of the fibers of the material to being cut or sheared off.



80. Ultimate Strength.—If samples of different materials are loaded until they break or rupture we can find out how much each material will stand. The *ultimate strength* of a material is the number of pounds per square inch of cross section of the material at which it breaks. This value varies between wide limits for different materials and often varies considerably for the same material.

The ultimate strength of a given material is different for the different stresses of tension, compression and shear. 81. Testing Machines.—For testing materials to find their ultimate strengths, testing machines as shown in the following figures are used.

Fig 191 shows a machine used for tension and com-

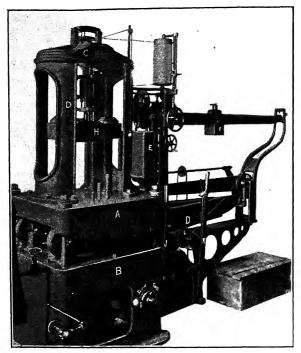


Fig. 191.

pression tests. The figure shows a piece being tested in tension.

The test specimen is placed in the machine as shown between heads (H) and (C) and is held firmly in place by means of wedge grips. Head (C) is fixed to the body

of the machine (A) by frame (D). The bed (A) rests on a set of levers. Through the head (H), two long screws are threaded which are geared to bevel gears shown inside of the case (B). A belt connected to a pulley which is fastened on a shaft to which the gears are fastened, drives the machine, which causes the screws to revolve slowly. This action moves (H) down causing a pull on the test piece and hence on the head (C), the pull being transmitted to (A).

(A) resting on lever (D) forces (D) down and by the system of levers, causes the lever on which (F) rests to move up. The system of levers is kept balanced by

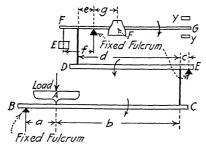


Fig. 192.

moving the weight (F) out along the graduated bar. (E) and (M) are weights used for balancing and for adjustment. All readings of the load applied and also the breaking load, are registered by means of (F), the lever on which it moves being marked off in pounds so as to read directly the pull exerted on the test piece.

When a piece is to be tested in compression, it is placed between the head (H) and (A) and it can be seen that as the head (H) moves down, the piece is compressed and the load registered as before.

The system of levers is shown in Fig. 192 giving an outline of their arrangement.

As the load is applied to the test specimen, it pushes down on (A) and tends to move lever (BC) around the fixed point at (B). The force at (C) then pulls down on the lever (DE) which is pivoted at (E). The lever (FG) is connected to the lever (DE) by the link (FD),

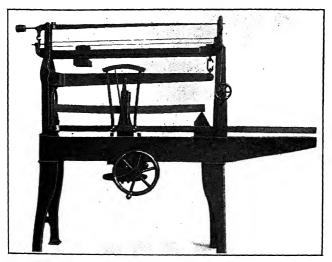
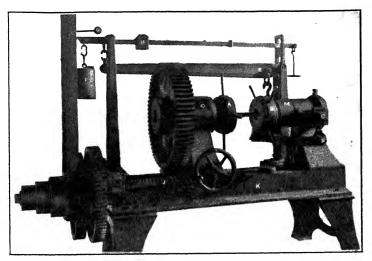


Fig. 193.

and as the force at (D) pulls down on the end (F) of lever (FG), the end (G) tends to move up. The end (G) is kept balanced between (Y) and (Y) by moving the weight (F) out along the lever. The object of the system of levers is merely to reduce the large load acting at (A) to a small load acting at (F). Nearly all testing machines are built on the principle of a system

of levers and the methods are worked out in a manner practically similar to that just described.

For bending tests of wood specimens a machine similar to that shown in Fig. 193 is used. The lever arrangement is similar to that of the tension machine. The method of supporting the specimen and recording the load on the beam and the deflection of the beam can be readily seen. This machine is operated by hand.



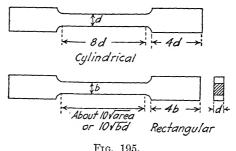
Frg. 194.

Pieces to be tested in torsion are placed in a machine similar to that shown in Fig. 194. The specimen is fastened in grips (A) and (B). (B) is fastened to (M) which has an arm (C) attached to it. By means of the belt shown at the left and the cone pulley and system of gears, (A) can be rotated. As it moves it tends to turn (B) and (M) and (C) tends to rise. Arm (C) then

pulls up on (D) which is a lever pivoted to the frame and so that the force is carried to the lever on which (H) rides. The lever is kept balanced by moving weight (H). The weight (F) serves as a balance. The system of levers here shown is practically the same as for the machine shown in Fig. 193 and the method of recording the load is similar.

82. Standard Test Pieces.—Figs. 195, 196 and 197 show a few of the standard test pieces or specimens used in finding the strength of materials.

The specimen used should in any case be as near an average specimen of the material as possible. If a



110. 155.

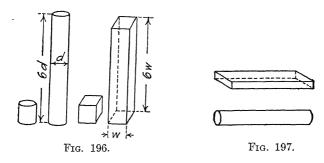
shipment of material is to be purchased, a few samples should be taken from different parts of the supply.

Fig. 195 shows a cylindrical and rectangular section of iron or steel as used in a *tension* test. The pieces are machined all over and filed along the middle part, for the body of the material should be as free from tool marks as possible and of a uniform size. The proportion of the parts is given in terms of the cross section of the material.

The blocks shown in Fig. 196 represent about the

limit of the size of pieces used in compression tests. A piece should not be longer than 6 times its diameter, else a bending may occur along with the compression.

The forms used for bending and shearing tests are like those shown in Fig. 197, the form in each case depending on the kind of material and the result to be determined. For torsion tests, the specimens are like those used for tension tests.



83. Values of Ultimate Strengths.—From the results of tests with these machines the following table shows the average values of ultimate strengths of the most used materials and for the kind of stress indicated.

ULTIMATE STRENGTH IN POUNDS PER SQUARE INCH

Material	Tension .	Compression	Shear
Cast iron	20,000	90,000	20,000
Wrought iron	55,000	50,000	40,000
Average steel	60,000	70,000	50,000
Soft copper	25,000	40,000	20,000
Hard copper	40,000	55,000	40,000
Wood	10,000	8,000	3,000
	,	,	(across grain)

Material	Tension	Compressor	Shear
Granite		15,000	1,500
Limestone			1,400
Concrete		0 -00	1,000
Brick	 	5,000	1,000

The following results represent a fair average

It would be of course unsafe to use in actual service material under such conditions that it would be subjected to a stress near the ultimate stress of the material, since the piece might break at any moment. A piece of material put in actual service is therefore designed for a stress per square inch, which is much less than the ultimate stress. This stress is called the working stress or the safe working stress.

- 84. Factor of Safety.—The ultimate strength divided by the working stress is called the Factor of Safety. For example if a piece of steel breaks at 80,000 lb. per sq. in. and we use a factor of safety of 8, the working stress is only ½ of 80,000 lb. or 10,000 lb. per sq. in., and the design is worked out for the latter value. The factor of safety of a material varies according to the material used and the manner of loading. A much larger factor of safety is allowed for bridge members on which there is a varying load than for a building for example where the load is practically constant.
- 85. The Elastic Limit.—In designing structures such values of working stress are always taken that the material will not be stretched beyond its "elastic limit," that is will not be worked at a unit stress beyond which the material will not return to its original shape when the stress is removed. The Elastic Limit which should always be greater than the working stress may also be defined as that unit stress beyond which the deforma-

tions of the material increase in a faster ratio than the applied loads, and beyond which the material will acquire a permanent set on the removal of the load, and not return to its original shape.

86. Values of Safe Working Stresses.—Following are safe working stresses in pounds per square inch for the most commonly used materials. The safe working load for tension, compression and shear are abbreviated respectively as follows  $S_t$ ,  $S_c$ ,  $S_s$ .

SAFE WORKING STRESSES-POUNDS PER SQUARE INCH

Material	$\begin{array}{c} {\tt Tension} \\ {\tt S}\iota \end{array}$	Compression S <sub>c</sub>	$_{S_s}^{\rm Shear}$
Cast iron	4,000	12,000	3,000
Wrought iron	10,000	12,000	10,000
Steel castings	12,000	16,000	12,000
Timber	800	800	600
			(with grain)

From the above we have the rule, the safe load "W" which can be carried for the several kinds of stresses is as follows:

For tension 
$$W = \text{area in sq. in.} \times S_t$$
.  
For compression  $W = \text{area in sq. in.} \times S_c$ .  
For shear  $W = \text{area in sq. in.} \times S_s$ .

Or in general for all stresses

$$W = \text{area} \times S$$
 from which 
$$\text{area} = \frac{W}{S}$$

These mean that the total safe load equals the stress per square inch times the area of cross section of the piece in square inches. And also the area required by a piece equals the total load divided by the safe load per square inch.

From the calculated area, the length of a side of the cross section, or the diameter in case of a circle, and so forth may be found.

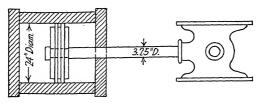
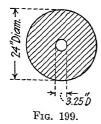


Fig. 198.

87. Strength of Rods.—When steam is applied to the back side of the piston shown in Fig. 198, the pressure puts the rod in tension. If the pressure is 180 lb. per sq. in. the total pressure on the piston is 180 times the net area of the piston, that is, subtracting the area of



which the pressure acts is shown cross sectioned in Fig. 199 and equals

(24)2 × 785 × (3.25)2 × 785 or

cross section of the rod. The area on

$$(24)^2 \times .785 - (3.25)^2 \times .785$$
 or 444.1 sq. in.

The total pull on the rod therefore equals

$$444.1 \times 180 \text{ or } 79,900 \text{ lb.}$$

that is, the applied load is 79,900 lb. and the total resisting stress is therefore 79,900 lb. The stress per sq. in. on the rod equals

$$\frac{\text{total load}}{\text{area of rod}}$$
 or  $\frac{79,900}{8.30}$ 

which equals 9630 lb., and this is well within the safe limit for steel. The unit compressive stress is also within the safe limit for steel. The compressive stress occurs when steam is admitted to the head end of the cylinder.

Example.—Find the size of a circular steel rod from which a weight of 4 tons is to be suspended.

Solution.—Taking 14,000 lb. per sq. in. as the safe working load in tension, we have

Area = 
$$\frac{W}{S}$$
 or  $\frac{4 \times 2000}{14.000}$  or .572 sq. in.

The diameter of the rod then equals

$$\sqrt{\frac{\text{area}}{.785}} \text{ or } \sqrt{\frac{.572}{.785}} \text{ or } .853 \text{ in.}$$

In figuring out threaded bolts the required area must be taken as that at the *root* of the threads, since a piece is no stronger than its weakest part. If the bolt has a sufficient strength for the area at the root of its thread, or in other words for its *smallest* area, the body of the bolt will have a surplus of strength on account of its greater area of cross section.

88. Strength of Ropes, Chains and Cables.—The ultimate strength of hemp rope is about 6000 lb. per sq. in. and for Manila rope about 3000 lb. per sq. in.

A safe working stress for hemp rope is 1400 lb. per sq. in. based on the *Nominal Area* of the rope, that is, assuming the area of cross section of the rope a solid circle. For Manila rope half the above value may be used.

The safe working load for chain links is given as 9000 lb. per sq. in. For iron cables the ultimate strength is about 40,000 lb. per sq. in. and for steel cables 80,000

1

lb. per sq. in. A factor of safety of 10 is often used for ropes and cables in finding the safe working stresses. This makes the safe value of working stress for iron cable 4000 lb. per sq. in. and for steel cable 8000 lb. per sq. in.

89. The Strength of Columns.—When a bar or rod subjected to compression has a length greater than 10 times its diameter it is called a *column*, and it must be worked out by difficult formulas which take account of its length, size and form of cross section. Columns may fail by buckling as well as being crushed, and most satisfactory results in some cases are obtained by testing the actual columns in a suitable testing machine. From the results of such tests the most reliable information on the strength of the columns is obtained.

#### **PROBLEMS**

1. It took 100,000 lb. pull in a testing machine to break a steel rod  $1\frac{1}{2}$  in. in diam. What was its ultimate tensional strength in lb. per sq. in.?

2. If a cast-iron bar  $1\frac{1}{2}$  in. by 2 in. in cross section breaks under a tensile load of 70,000 lb., what load in lb. per sq. in. would break a  $2\frac{1}{2}$ -in. diam. cast-iron rod of equal unit strength?

- 3. A round cast-iron bar is to be subjected to a tension of 34,000 lb. If it is designed so that the unit stress is 2500 lb. per sq. in., find the diameter of the rod.
- 4. A wooden block 15% in. square was crushed in a testing machine under a load of 21,640 lb. What was its ultimate compressive strength in lb. per sq. in.?
- 5. It required 87,630 lb. to shear off in a testing machine a  $1\frac{1}{4}$ -in. diam. steel rod. What was its ultimate shearing strength in lb. per sq. in.?
- 6. If the ultimate compressive stress of wrought iron is 55,000 lb. per sq. in. and it is subjected to a working stress of 11,500 lb. per sq. in., what is its factor of safety?
- 7. A 7/s-in. diam. wrought-iron tie rod on a bridge is subjected to a working stress of 9000 lb. per sq. in. What load is it sup-

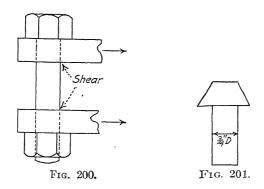
porting? If the ultimate stress is 50,000 lb. per sq. in., what factor of safety is used in its design?

8. Find the compression in lb. per sq. in. for a 3-in. diam. piston rod carrying an 18-in. diam. piston with a steam pressure of 180 lb. per sq. in.

9. Find the tensile stress in lb. per sq. in. for a 3½-in. diam. piston rod with steam at 175 lb. per sq. in. acting on the back of a 20-in. diam. piston attached to the rod.

(Note.—Subtract the cross-sectional area of the piston rod from the piston area to get the net area on which the steam acts.)

10. What is the stress per sq. in. in the studs holding on a cylinder head for a 24-in. diam. cylinder if there are twenty-six



 $\frac{7}{8}$ -in. diam. studs and the greatest steam pressure is 200 lb. per sq. in?

(Note.—The area at root of thread of each 7/8-in. stud is .419 sq. in.)

11. A solid cast-iron cylinder 2½ in. in diam. is under a compression of 40,000 lb. Find its factor of safety.

12. A brick 2 in.  $\times$  4 in.  $\times$  8 in. weighs about  $4\frac{1}{2}$  lb. Find the height of a pile of bricks so that the compressive stress per sq. in. on the lowest brick will be  $\frac{1}{2}$  of the ultimate strength.

13. A wrought-iron bolt as shown in Fig. 200 is to be subjected to shear in two places. If the load is 3000 lb. and the factor of safety 5, find the diameter of the bolt.

- 14. What size of hemp rope is required to lift a casting weighing 21/2 tons?
- 15. What is the total safe load which a steel cable 2 in. in diam. can stand?
- 16. For an ultimate tensile strength of 50,000 lb. per sq. in. find the strength of the 3/4-in. diam. cone rivet shown in Fig. 201.

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